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Particle Order: A New Fundamental Concept in Hadron Physics

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PART TWO: THE GENERALIZED HADRONIC *S* MATRIX

7. BARYONS AND GENERALIZED ORDER

7.1. The Need for an Ordered *S*-Matrix Scheme that Includes Baryons

The sequentially ordered SM, and the DTU scheme based on it, were seen to describe from a unified point of view a wide range of phenomena in the meson sector; indeed, it appears to have the potential of becoming a quantitative theory of mesons (of course a strong-interaction theory cannot be complete without including all hadrons, but to the extent that baryons do not affect the calculations too strongly, it appears to be a good approximation for mesons).

Given this state of affairs, we found it hard to believe that the ordered SM idea would remain limited to mesons, particularly since baryons, too, exhibit features that we had come to recognize as the hallmark of the ordered SM, such as quark structure (qqq), the OZI rule (e.g., $\psi \to p\bar{p}$ etc. is strongly suppressed), Regge pole dominance, exchange degeneracy for many trajectories, etc.

In fact, the need to extend duality and particularly the DTU program to baryons had been felt acutely for quite a while, and had given rise to numerous attempts to accomplish this, none of them really successful.

One of the more persistent obstacles was the problem posed by the

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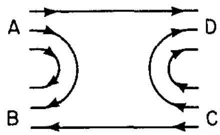
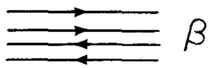
exotic mesons often called baryonium. Duality demands that the four-baryon amplitude

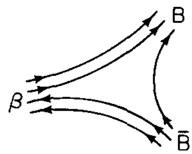


be expressible as a sum over baryonium resonances in the s_{AD} channel.

Thus baryonium, which one denoted by



coupled directly with the baryon–antibaryon channel:



On the other hand, it was known from the properties of purely mesonic planar amplitudes that mesonic channels did not couple to baryonium. The problem that the above-mentioned theories tended to encounter was that baryonium coupled to mesonic channels even at the planar level; this also conflicted with the reality of the OZI rule.

In recent years baryonium candidates have been found experimentally, some apparently even with exotic quantum numbers. And, in fact, they prefer to decay into baryon–antibaryon pairs rather than into mesons in spite of the far smaller phase space available for the former decay, indicating that a planar SM approximation in which baryonium is prohibited from decaying into nonexotic mesons is desirable from the experimental point of view, too.

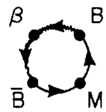
7.2. The Failure of Sequential Order to Accommodate Baryons

Sequential order is characterized by "twoness": each particle has two neighbors, a predecessor and a successor. This twoness was seen to manifest itself in the $(q\bar{q})$ nature of mesons. So the fact that in the conventional quark models baryons are characterized by "threeness" (qqq) already indicates quite clearly that sequential order alone is not going to accommodate baryons.

Nevertheless, we first tried to include baryons in a sequential scheme in two different ways before we saw ourselves forced to abandon the attempt.

One of the schemes, the "diquark" model, tried to fit baryons into the framework of sequentially ordered amplitudes without any modification.

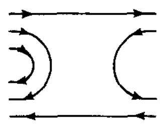
One simply postulated the existence of certain "baryon flavors," denoted by a wavy quark line. For example, in



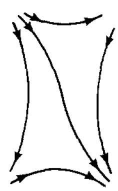
β stands for baryonium, B for a baryon, $\bar{B}$ for an antibaryon, and M for a meson. In order to regain the conventional notation, one replaced

by

(hence the term "diquark" for the wavy quark line). This scheme failed completely. Besides being asymmetrical and ad hoc, it yielded the wrong spectrum. Furthermore, many processes like $pK^- \to \Xi^- K^+$ could not be expressed as diquark processes

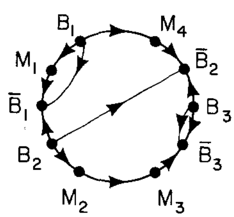


but only as

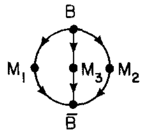


so that the corresponding planar amplitude was forced to be zero; and yet experimentally these processes were not suppressed compared with processes that could be written as diquark processes. For all these reasons and others this scheme was rapidly abandoned.

The other scheme, while still sequential in a sense, already required a broadening of this concept. It allowed for nonadjacent baryon–antibaryon pairs connected by a "mate line." Mate lines were not allowed to cross, and



were required to be "dead," i.e., no other particles such as mesons were allowed on them; e.g.,

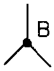


is forbidden. This asymmetry of mate lines with respect to the other quark-lines was the price to be paid for the preservation of a well-defined sequential order, but it was also the main drawback of the scheme, leading to various undesired consequences. Besides, it was decidedly artificial and ugly.

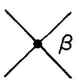
If we abandon the "deadness" of the mate lines and with it sequential order, then we arrive, by minimal generalization of sequential ordering, at the same scheme that we will in the following derive by the converse process: namely, starting from the most general order, we will use consistency requirements imposed by general SMT to exclude inconsistent types of order, and thus arrive at our theory by specialization (from the general to the special).

7.3. The Key to Baryons and Other Hadrons: Generalized Order

We now take the hint offered by conventional quark diagrams and represent baryons by three-vertices



while mesons continue to be represented by two-vertices as before. The example of baryonium indicates that there will be four-vertex particles

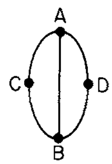


in the theory, too. Indeed, as we will see, the presence of three-vertex particles in conjunction with the pole conjecture implies the existence of n-vertex particles for every integer $n \geqslant 2$; the number of edges emanating from a particle vertex is of course a characteristic property of that particle.

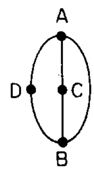
Once we include particles other than nonexotic mesons, as we now wish to do, we have to *generalize the concept of order* to accommodate them; sequential order can no longer achieve this. The most general kind of order amongst a set of objects is represented by a *connected graph* whose vertices represent the objects, and whose connectivity structure represents the order.

We exclude the possibility of "tadpoles," i.e., of a vertex connected to itself by means of an edge

Note that a graph is *topologically* defined: it is only the connections between the edges that count, and not the particular way that the graph is drawn on the plane; e.g., the two graphs

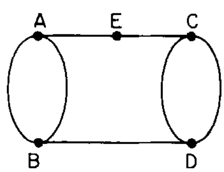


and

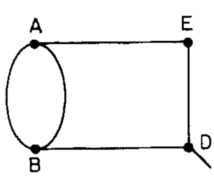


are considered identical.

We now postulate that every hadronic scattering process is ordered in the above, generalized sense: it is characterized, besides by the individual particle parameters (t_i, p_i, μ_i), by an order represented by a *process graph* whose vertices represent the particles, each with its characteristic number of edges emanating from it. All edges in a graph have to be "saturated." That is, if a particle (such as a baryon) corresponds to a three-vertex, then that three-vertex actually has to be connected with three other particles (not necessarily distinct);

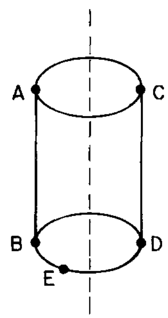


is okay, whereas

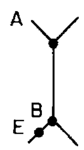


is not, because it has an unsaturated edge. To each such ordered process corresponds an *ordered amplitude* expressing the probability amplitude for that process to happen, in the usual sense.

Any bisection of a process graph into two connected subgraphs defines two *ordered channels*, graphically represented by the subgraphs that we call *channel graphs*. For example, the bisection

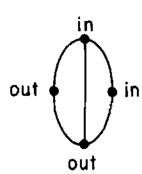


yields the channel graphs

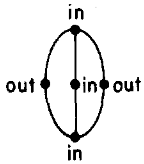


and

When both the "in" particles by themselves and the "out" particles by themselves form a connected subgraph and hence define a bisection of the process graph, then we call that process an *ordered transition*. [We will start out by regarding ordered transitions only; the amplitudes for all other ordered processes can be obtained from ordered transition amplitudes by crossing, as in the sequentially ordered case.] For example,



is an ordered transition, whereas



is not.

All this is completely analogous to the sequentially ordered processes; the only difference is that we allow a more general kind of order now.

So far we have only established the general outlines of the ordered SM framework. As it stands, it is far too general. What kinds of particles, of all the possible ones, actually occur? What kinds of graphs, of all the possible ones, can actually occur as process graphs? Which ordered channels communicate, or in other words, what characterizes an ordered sector? Is it possible to define a unitary, cluster-decomposable analytic ordered SM with pole factorization and crossing and the other properties of Part One, Section 3, in the context of generalized order?

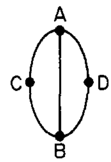
It is these questions that we will examine in Section 9. And we will see that the general requirements of SMT and generalized order tend to contradict one another; they can be made compatible only under special circumstances: if we introduce "color," and restrict ourselves to certain kinds of particles and certain categories of process graphs. Thus the implementation of consistency between the general principles of SMT and order yields an "order bootstrap," the result of which is a *specific* theory that generalizes the sequentially ordered SMT of Part One. It continues to satisfy all the properties of Part One, Section 3, including duality, and predicts a zero-triality quark spectrum as observed in nature.

Before we proceed with this, however, we wish to insert a section with all the concepts and results from graph theory that we will be needing in this work.

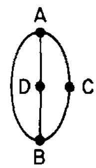
8. DEFINITIONS AND FACTS FROM GRAPH THEORY

8.1. Some General Concepts

Graphs. A graph is a set of n vertices, some pairs of which are connected by lines. These lines are called *edges.* Every edge starts on a vertex and ends on another vertex. If one can pass from any vertex to any other vertex of the graph by moving along edges, the graph is called *connected.* A connected graph represents an *order* amongst its vertices. This order is *topological* in that it depends only on which of the vertices are connected ("neighbors"). Two graphs are considered equivalent if they consist of the same vertices, connected in the same way (even if they are drawn on the plane in distinct ways, e.g.,



is equivalent to



A two-vertex

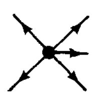
is often called a *trivial vertex* because its omission does not affect the connectivity structure of the remaining vertices of the graph. A one-vertex

is called *pendant.*

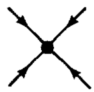
When a connected graph is cut into two connected subgraphs we call the cut a *bisection.* The subgraphs are strictly speaking not graphs because the *free edges* (that were cut by the bisection) do not end in a vertex; nevertheless we call them graphs here (we refer to them as channel graphs). Two channel graphs obtained by bisection of a graph G are called *cographs* with respect to G.

A *directed edge* starts on one vertex and ends on another; this is denoted by an arrow:

A graph all of whose edges are directed is called an *oriented graph.* A vertex with the property that all edges impinging on it are either directed towards it or away from it, e.g.,

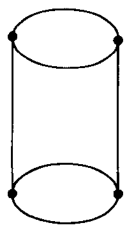


or

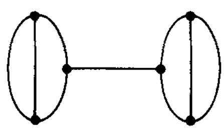


is called an *oriented vertex*. A graph whose edges can be directed in such a way that all vertices are oriented is called vertex orientable or *bipartite*. A graph is bipartite if and only if the vertices can be classified into two sets in such a way that any vertex from one class is directly connected only to vertices from the other class. And this is the case if and only if every closed loop of the graph contains an even number of oriented vertices.

A graph is called *n-colorable* if and only if each edge can be assigned one of n (and not less than n) colors in such a way that no two adjacent edges (i.e., edges that come together at a vertex) are assigned the same color. For example,



is 3-colorable, whereas

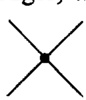


is not; however, it is 4-colorable.

A graph is called *planar* if it can be drawn on a plane (or sphere) without any crossing of lines. A graph is called *cubic* if it contains only three vertices.

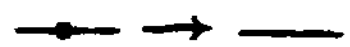
Given a channel graph, such as

we can *contract* away all internal edges, exhibiting only the free edges



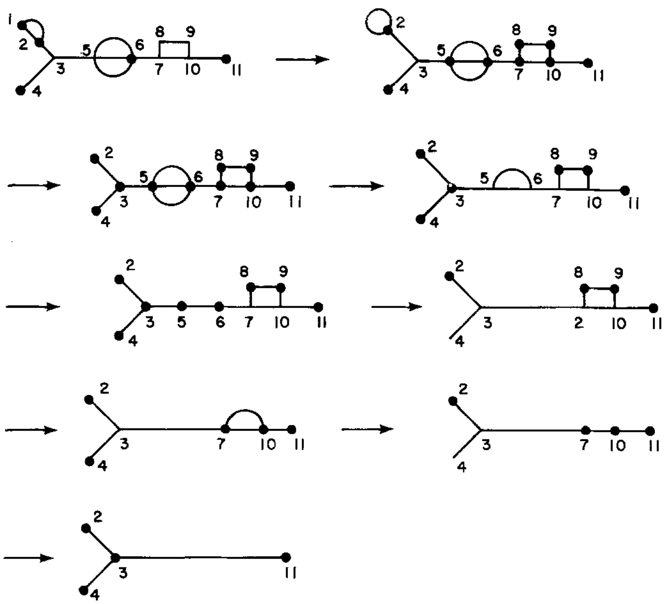
The converse operation is called *expansion*. For example, any n vertex with $n \geqslant 3$ can be expanded into a cubic tree graph. This fact exhibits the special role of the three-vertex in graph theory and hence presumably the reason for the existence of three colors; for the statement "any vertex can be expanded into a tree graph consisting of n vertices only" is true only for $n = 3$.

An important concept is that of the spanning tree of a graph. Given a graph G, if we successively eliminate all trivial vertices



and all tadpole edges

then we obtain a tree graph that is called a spanning tree of G. In general there will be several spanning trees for one and the same graph, depending on the order in which edges are eliminated. Graphs that have only one spanning tree are called *uniarboral*. For example, if $G =$



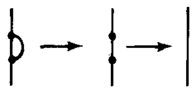
we get the same spanning tree for any order of reduction. Hence, G is uniarboral.

The set of all spanning trees of a graph, its so-called *spanning forest*, characterizes the graph except for the positions of trivial vertices

and "necklaces"

both of which do not affect the connectedness structure. In other words, given the spanning forest of a graph, we can reconstruct the graph itself uniquely except for trivial vertices and necklaces, by simple superposition, uniarboral channel graphs are going to play a crucial role in ordered SMT.

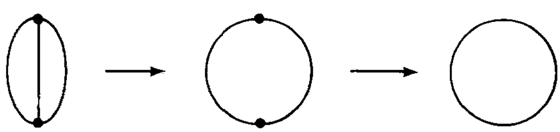
A somewhat related, but more specialized, concept is that of *reduction*. If in a graph G any two vertices are connected by more than one edge, then if we erase all but one of these edges and eliminate the resulting trivial vertices, we are said to have performed a reduction:



When a graph can in a finite series of such reductions be reduced to a circle, it is called *reducible*. Conversely, of course, every reducible graph can be constructed starting from a circle and successively replacing edges

with "necklaces"

Examples of reducible graphs are



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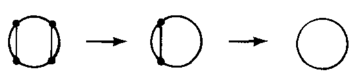
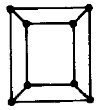


or

On the other hand,



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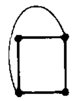


are not reducible.

Since cubic reducible graphs are going to play a central role in ordered SMT, we now examine their properties more closely.

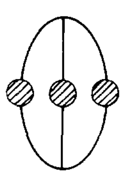
8.2. Cubic Reducible Graphs and their Properties

Mate Structure. In a cubic reducible graph (CRG), there exists to every vertex V another, uniquely defined vertex $\bar{V}$, called its *mate*, with which V is contracted in the course of the reduction. It is easy to see that the vertex $\bar{V}$ with which a given vertex V is contracted away in the reduction does not depend on the order in which the steps of the reduction are performed; hence the mate is well defined.

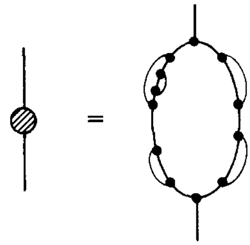
If, in particular, for given vertex V, we reduce away all the vertices that can be reduced while leaving V intact, then we arrive at the graph

Obviously, if V_2 is the mate of V_1, then V_1 is the mate of V_2: $\bar{\bar{V}} = V$. We call $(V, \bar{V})$ a *mate pair.*

For any mate pair $(V, \bar{V})$ of a cubic reducible graph G, G can be represented in the form



where the blobs stand for subgraphs that can be reduced down to a line, e.g.,

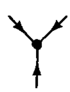


From this representation it is easy to see that every closed path starting out from a vertex V by one edge and finally returning to V along one of the other two edges has to pass through $\bar{V}$; and furthermore, $\bar{V}$ is the only vertex that has this property with respect to V. Conversely, this property characterizes reducible graphs.

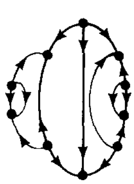
Bipartiteness. By construction every closed loop of a CRG contains an even number of vertices; hence we can divide the set of vertices into two subsets B and $\bar{B}$ in such a way that a vertex from B is only connected to vertices from $\bar{B}$, and vice versa: thus all CRGs are bipartite. If a vertex $V \in B$, then its mate $\bar{V} \in \bar{B}$; and if $V \in \bar{B}$, then $\bar{V} \in B$. From this it immediately follows that the number of vertices in B equals the number of vertices in $\bar{B}$ for every CRG: $\#(B) = \#(\bar{B})$.

Since bipartiteness is equivalent to vertex orientability, every CRG can be oriented (i.e., an arrow attached to every edge) in such a way that all vertices are oriented, either

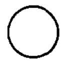
or



The former vertices by convention form B, the latter $\bar{B}$, e.g.,

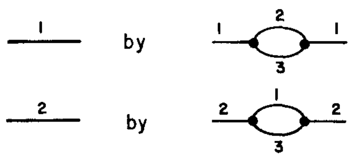


Three-Colorability. It is easy to see that a CRG is always three-colorable: such a graph can be constructed from a circle

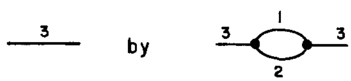


by successively substituting

so we color this circle with one of three colors, and then substitute



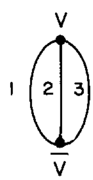
and



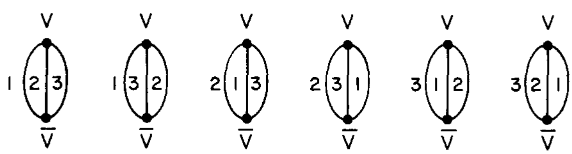
successively until the desired CRG is constructed: it is then by construction consistently colored with three colors such that no two adjacent edges have the same color.

Planarity; Spherical Representability. By construction it is clear that a CRG is planar: it can be drawn on a sphere or plane without crossing of edges.

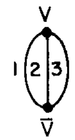
Regard a three-colored CRG. Then there are various different ways that it can be drawn on a sphere, all corresponding to the same graph. For example,



can be drawn on a plane in the following distinct ways:



On a sphere there are only two different representations, namely,



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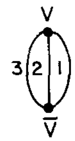
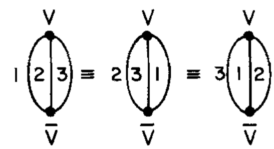
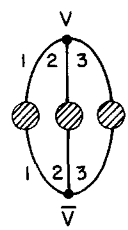


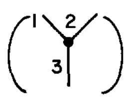
since, e.g.,



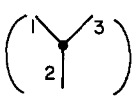
We can use this redundancy of the spherical representation to express nearly all the color information of the CRG. We first note from the representation



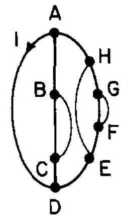
that if a vertex V is clockwise oriented



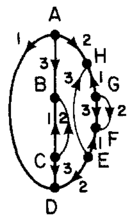
then its mate $\bar{V}$ is counterclockwise oriented:



Otherwise there is no limitation on the relative orientations of vertices: thus in drawing the graph of a sphere we can, e.g., freely choose the clockwise (or, respectively, counterclockwise) orientation of every B-vertex (the orientation of the $\bar{B}$ vertices is then automatically determined). We can therefore by convention always draw a colored CRG on a sphere in such a way that all B vertices are clockwise, all $\bar{B}$ vertices counterclockwise oriented. If we follow this convention then we need only specify the color of one single edge of the whole graph (and its direction) in order to know the color (and direction) of every edge of the graph. For example,



represents

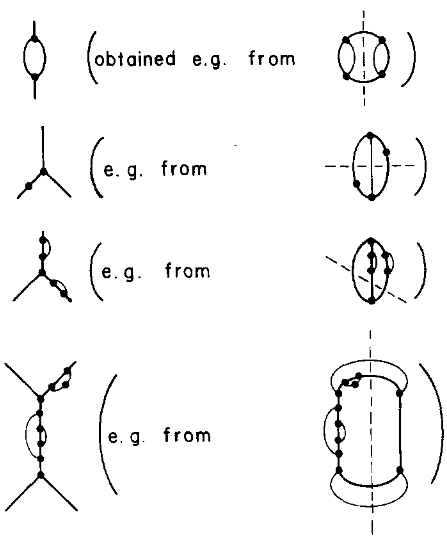


We will not make use of this spherical representation in this work, although it played quite an important role in the evolution of our scheme and is useful for the topological expansion (Sursock, 1978).

8.3. Cubic Reducible Channel Graphs

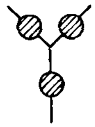
When we perform a bisection upon a cubic reducible graph, the resulting two subgraphs are called cubic reducible channel graphs (CRCG).

Examples of CRCGs:

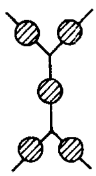


It is apparent from these and other examples, and can easily be shown generally, that every CRCG has the form of a tree graph with reducible "blobs" inserted on the internal and free edges. For example, the first example above is of the form

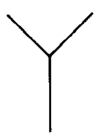
the second and third of the kind



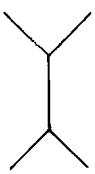
and the fourth of the form



We say that the first CRCG has the *skeleton* |, the second and third have the skeleton



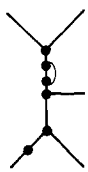
the fourth one the skeleton



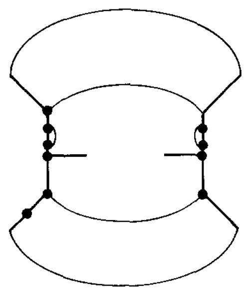
Since for the reducible graphs the operations of reduction and of constructing the spanning tree are identical, the skeleton of a CRCG coincides with its spanning tree. CRCGs are uniarboral; indeed, the property that any bisection of the graph leads to subgraphs that are uniarboral *characterizes* reducible graphs; this fact underlies the physical importance of reducible graphs.

As we saw above, the skeleton of a CRCG is a cubic tree graph obtained by complete reduction of the CRCG. The number of vertices of the skeleton equals the number of mate pairs that were separated by the bisection that generated the channel graph in question (all the unseparated mate pairs get eliminated in the reduction); the number of free edges of a CRCG, and hence of its skeleton, is $n + 2$ if the number of vertices of the skeleton is n.

We have seen that every CRCG can be transformed by reduction into a tree graph, its skeleton. Conversely, every channel graph with this property can be obtained by bisection of an appropriate CRG, and is thus by definition a CRCG. The simplest way to see this is to take the channel graph in question, e.g.,



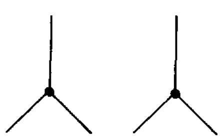
take another, identical one and then join the two by their corresponding free edges; e.g.,



the result is a CRG.

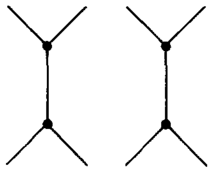
The set of all CRCGs can be categorized according to their skeletons into *skeleton classes.*

Two CRCGs obtained from a CRG by bisection always belong to the same skeleton class. Thus two CRCGs from different skeleton classes can never be joined to form a CRG. On the other hand, as we saw above, two CRCGs from the same skeleton class can always be joined to a CRG. This joining can in general be performed in more than one way. For example, the two CRGs

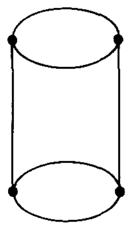


can be joined up in 3! = 6 different ways to the CRG

and the two CRGs

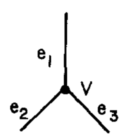


can be joined up in eight different ways to the CRG

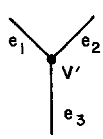


As a result the *resewing* of two CRCGs obtained by bisection of a CRG is not unique: given the two CRCGs, we have no way of knowing in which way they were joined before the bisection, and so we cannot resew them to the original CRG.

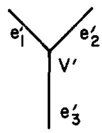
A little reflection shows that the source of this ambiguity is the presence of nontrivial automorphisms on a CRCG. An automorphism on a graph is a mapping that maps each vertex of the graph into a vertex of the graph and each edge into an edge in such a way that the topological relations are preserved: if and only if a vertex V lies on an edge e does the image of V lie on the image of e. Thus, e.g., the CRCG



has six automorphisms: $V \to V$, $e_1 \to e_1$, $e_2 \to e_2$, $e_3 \to e_3$ (the trivial automorphism); $V \to V$, $e_1 \to e_1$, $e_2 \to e_3$, $e_3 \to e_2$, $V \to V$, $e_1 \to e_2$, $e_2 \to e_3$, $e_3 \to e_1, \ldots$, etc. It is the presence of these six automorphisms that leads to the sixfold ambiguity of resewing



with

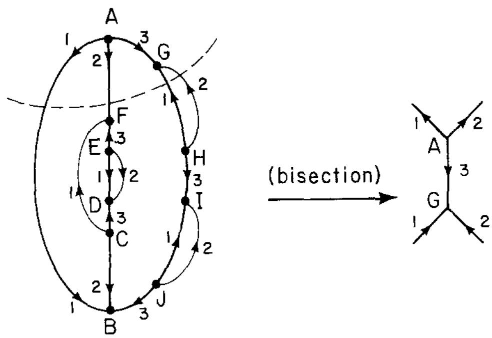


And similarly with the other example above.

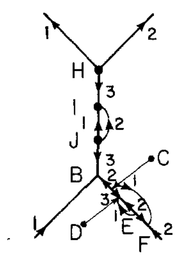
Three-Colored, Vertex-Oriented CRGs. We mentioned before that CRGs can be vertex oriented and also three-colored. We now examine some properties of vertex-oriented and three-colored CRGs.

Every bisection of such a graph yields two vertex-oriented, three-colored CRGs, each with a skeleton that is now also vertex oriented and three, colored; these two skeletons are identical in topological structure and color. but they have opposite vertex orientation. They are called *conjugate* skeletons.

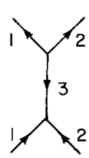
As an example we have



and



with the skeletons



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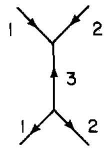
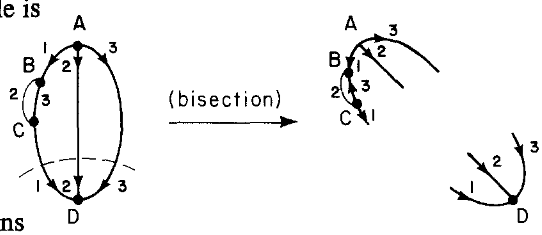
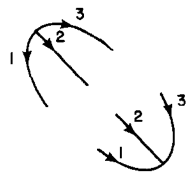


Another example is

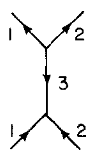


with the skeletons



and

Note that the skeletons



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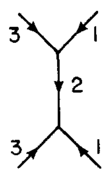
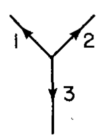
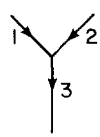


are distinct (color), as are



and



(orientation: the latter pair is conjugate).

Two vertex-oriented, three-colored CRCGs can be joined to form a vertex-oriented, three-colored CRG if and only if their skeletons are conjugate. But, unlike before, they can be joined *only in one unique way*! This fact will be of crucial importance when we apply all this to physics. It implies that if we bisect a vertex-oriented, three-colored CRG, then the resulting channel graphs can be resewn in exactly one way to a CRG (namely, the original one). The reason for this is that for a given vertex-oriented, three-colored CRCG there is no automorphism but the trivial one (note that now an automorphism is also required to map an edge of any color into an edge of the same color, and if an edge e begins (ends) on a vertex V, then the image of e must begin (end) on the image of V).

9. THE ORDER BOOTSTRAP: CONSTRUCTION OF THE GENERALIZED ORDERED S-MATRIX THEORY

9.1. Some General Features of S-Matrix Theory and the Restrictions they Impose upon the Possible Forms of Order

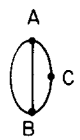
In Section 7 we set up the general framework of ordered SMT, but we remarked at the end that it was too general and required a definite form, i.e., a specification of which graphs represented possible structures of particle order, and of the nature of the particles themselves and of the sectors they formed. This we are now about to do. The tool we want to use to achieve it is compatibility of order with the general features of SMT. As we will see, considerations of this type will suffice to determine the form of the theory completely.

9.1.1. Unique Resewing. In Section 3.2 of Part One, we have already explained that in an ordered SMT the initial channel graph and the final channel graph should determine a definite process graph. Since this implies that two channel graphs obtained from bisection of a process graph should be resewable in a unique way, we called this the criterion of "unique resewability." Already there we saw that this criterion necessitated the orientation of graph edges in order to construct a consistent sequential theory. Here, for generalized order, the implications are even further reaching.

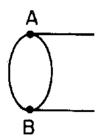
Since the problem of resewing boils down to which pairs of free edges of the two channel graphs should be joined, the criterion of unique resewability can be formulated in terms of an appropriate labeling procedure for the free edges of a channel graph. In order to determine the unique and correct way of matching up the free edges of the two channel graphs, the labeling of the free edges must have the following properties:

(i) The labeling must be determined by the structure of the channel graph itself, without reference to the co-channel graph with which it formed the process graph before bisection.
(ii) The labeling must be such that each free edge receives a distinct label distinguishing it from all other free edges.
(iii) Two channel graphs obtained by bisection of a process graph should receive identical labels on free edges that were joined together.

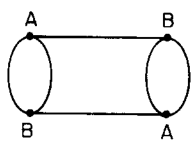
9.1.2. Closure of the Set of Channel Graphs under Bisection and Resewing. Given a legal process graph G (i.e., one with a nonzero ordered amplitude), then every bisection of G yields two legal channel graphs (corresponding to interacting ordered channels). Each of these can be sewn together with any communicating channel graph, and in particular always with one identical to itself to form a new legal process graph; this in turn can be bisected in every possible way to form a new set of legal process graphs, etc., etc. The set of legal process graphs must be closed under the operations of bisection and resewing. For example, if



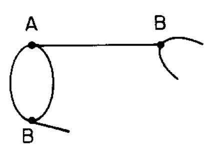
is legal, then



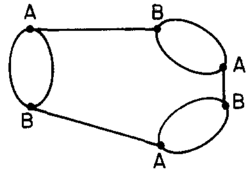
is a legal channel graph, and hence



a legal process graph, and hence



is legal, and hence



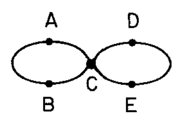
is legal, etc.

9.1.3. The Particle–Pole Identity and Some of its Consequences: Composability, Coconnectedness, Confinement. In SMT every external particle t of a scattering process A may be regarded as an internal particle (pole) connecting A with another scattering process B to a joint (double-scattering) process $(A \overset{t}{\text{—}} B)$.

In ordered SMT all these processes A, B, $A \overset{t}{\text{—}} B$, are ordered. The order of $(A \overset{t}{\text{—}} B)$ is obtained from that of A and B by bisecting out the particle t from the process graphs of A and B and then sewing together the resulting channel graphs (according to Section 9.1.1 this can be done in a unique way).

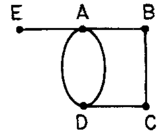
A consequence of this is the *composability* of process graphs: any two (legal) process graphs that both contain a given particle A can be composed to a larger process graph by erasing that particle vertex A in both process graphs and joining the corresponding free edges. The same is true if we replace the word "particle A" by "ordered channel."

Another consequence of the particle–pole identity is that every legal process graph must be *coconnected*: it must have the property that erasing any particle graph leaves the remainder of the graph connected; i.e., each particle corresponds to a bisection (this is because poles occur only in channel variables corresponding to bisections of the process graph, as we saw in Part One). This eliminates graphs like



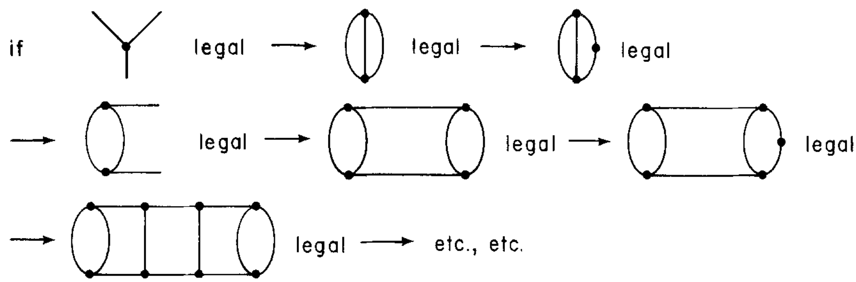
that are held together by one single particle. It also eliminates the possibility of particles corresponding to one-vertices (pendant vertices), except possibly in trivial process graphs like

This is because the particle to which the pendant vertex is attached cannot be erased without disconnecting the pendant vertex from the rest of the graph, e.g.,



(A holds graph together). Since pendant vertices would, if they existed, be identified with free quarks (as we shall see), this implies the "confinement" of quarks.

9.1.4. Triviality of the Two-Vertex. In graph theory a two-vertex is considered trivial and can be omitted or inserted on any edge without altering the topological structure of the graph. We therefore assume that if we omit or insert appropriate trivial vertices on the edges of legal process graphs we again obtain legal process graphs. This leads to a strengthening of the restriction imposed by Section 9.1.2, e.g.,



9.1.5. Particle–Sector Correspondence; the Pole Conjecture. We noted in Part One the close correspondence between particles and sectors: to every particle corresponds a unique sector (set of communicating channels) with which it communicates; conversely, every sector contains a set of particles (according to the pole conjecture this set is nonempty), differing from one another only by their space-time quantum numbers m, s, P, but with identical graphical representations and flavor structure. This set of particles is characteristic of the sector.

So far we have left somewhat vague the question as to how particles are to be graphically represented in the theory, maybe raising the impression that they can all be represented as vertices. However, we now note that although that may be true for some particles (the mesons of Part One were one example; baryons and antibaryons will be shown to be another), the more general statement is that particles should be graphically represented in the same way as sectors. If, e.g., all ordered channels with n free edges communicated with one another, then a sector would be completely characterized by how many free edges the channel has; under these circumstances, and only under these, could a particle with n free edges be represented as a (structureless) n-vertex. But we will soon see that this is not a consistent assumption, and that particles with more than three free edges cannot be represented as simple vertices, but instead are represented by tree-graphs, because sectors are represented by tree graphs (spanning trees).

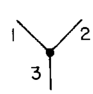
For now, we just note that the problem of particle representation is tied up with the problem of sector representation in a circular way: in order to even define channel graphs and sector graphs, we need the particle graphs out of which they are constructed; but we only know the particle graphs once we have worked out the way that sectors are to be represented. We work our way out of this dilemma by starting our construction only with particles that can definitely not be represented in any other way than as simple vertices, namely, two-vertex particles and three-vertex particles (so-called nonexotic particles). Having built up the theory with these particles alone, and worked out the rules governing sectors and their representations, all the other particles are delivered into our hands effortlessly.

9.2. The Order Bootstrap

9.2.1. Color. Let us regard the simplest nontrivial vertex, the three-vertex; unlike higher vertices, it cannot be expanded into simpler vertices (see Section 2.1). Let us assume there exists a particle with the particle graph

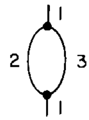
As we saw, the criterion of unique resewability demands that all the three free edges of this graph be distinguishable; they therefore have to be labeled, and this label is an intrinsic property of the edge. We call this label *topological color* or just *color*. We wish to stress that we are using this term in the sense of graph theory (as in "four-color-theorem"), and not in the sense of QCD; although the concepts will turn out to be related, they are not identical.

Thus the edges of every three-vertex particle graph receive a distinct color label:

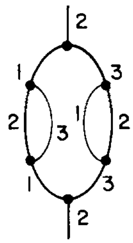


in a process graph containing only such particles all edges are colored in such a way that no adjacent edges have the same color. When we resew two channel graphs to a process graph, only free edges of the same color can be joined.

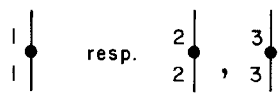
What about two-vertex particles? Any two-vertex particle that is generated as a pole in an ordered channel consisting of three-vertex particles, e.g.,



or



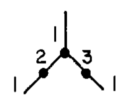
is obviously of the kind



with both edges of the same color.

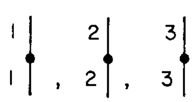
But could there exist two-vertex particles with two differently colored edges, like

The answer is no; for if there were, then channel graphs like

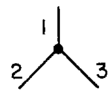


would exist, and with them (pole-conjecture) particles like

whose edges are not distinguishable, which would defeat the very purpose for which color was introduced. Thus two-vertex particles have to be of the kind mentioned before:



and by the pole conjecture these particles certainly exist if the three-vertex particles



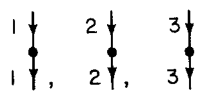
exist.

9.2.2. The Orientation of Edges and Vertices. Now, however, we seem to be in serious trouble, for two-vertex particles

etc., necessarily occur in the theory, but apparently violate the criterion of unique resewability, since the two free edges cannot be distinguished.

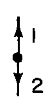
But there is a solution to this dilemma, a solution that is immediately suggested when we recall that this generalized ordered SMT is to be a generalization of the sequentially ordered SMT. For there we saw that edge orientation is a necessary feature of the theory, providing the distinction between the two edges of a particle that is necessary for unique resewability: a two-vertex particle is necessarily of the type

We therefore postulate that all edges are oriented, and that two-vertex particles are of the nature



They indeed merit the name "trivial vertices" since their insertion or omission alters neither the orientation nor the color of an edge. We will call these particles (nonexotic) mesons. Particles of the type

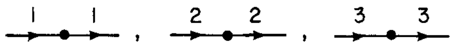
are forbidden for the same reasons as we introduced edge orientation before. And particles like



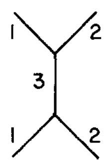
would, through the formation of a channel such as

lead to the existence of the forbidden particles

and are thus also eliminated. The only allowed two-vertex particles are thus the above trivial vertices

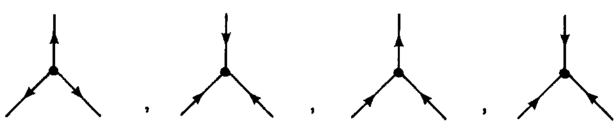


We have deduced the necessity for edge orientation by regarding mesons. We could just as well have proceeded by looking at channel graphs of the kind

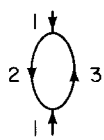


Here again the demand of unique resewability leads to the necessity of orienting the edges, this time directly in terms of the free edges of three-vertex particles, because there is again a nontrivial automorphism, mapping the two vertices into one another. Similar considerations can be invoked to confirm that indeed *all* edges have to be oriented, not only those of two-vertex or three-vertex particles, just as all edges have to be colored.

A priori there could be four types of three-vertices:



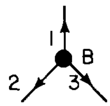
But if vertices of the three type existed, it would lead to channels of the type



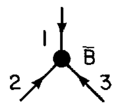
and hence of particles

of the forbidden kind; and the fourth type of vertex would similarly lead to forbidden particles like

Therefore only three-vertices of the first two kinds can exist. We call

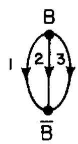


a *baryon vertex*, and

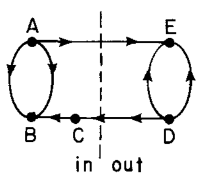


an *antibaryon vertex*, and the corresponding particles baryons and antibaryons.

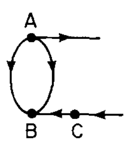
Obviously, as the trivial process graph



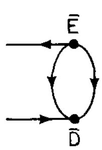
shows, antibaryons are the antiparticles of baryons: it describes a baryon B going into itself. More generally, charge conjugation of an ordered state converts the channel graph into the *conjugate channel graph*, obtained by reversing the orientation of all the edges. In the bra–ket formalism, a process graph like



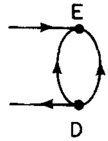
refers to a transition from a bra with a channel graph



to a ket with channel graph

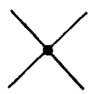


i.e., the *conjugate* of the channel graph

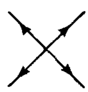


obtained by bisection of the process graph. This is again just as for the sequentially ordered SM, so we do not elaborate any further.

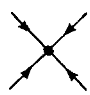
9.2.3. The Nonexistence of Simple n Vertices for $n > 3$: All Process Graphs Are Cubic. The next question we ask is whether the existence of particles corresponding to simple n vertices, with $n > 3$, is compatible with the existence of baryons. Let us assume that such a particle existed, say for $n = 4$. Its particle graph would then be



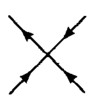
where the edges still have to be colored and oriented. We first note that for the same reasons as for three vertices, all the edges have to be oriented towards or away from the vertex: either



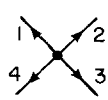
or



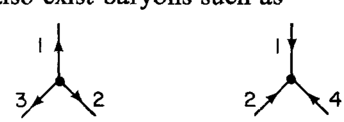
but not, e.g.,



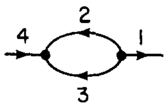
Therefore, in order to distinguish the edges, we need four colors.



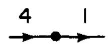
But then there would also exist baryons such as



and hence ordered channels like

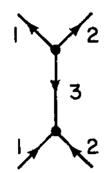


As a result there would have to exist particles like



which are forbidden. Thus the existence of more than three colors is incompatible with the existence of baryons, and so simple n vertices with $n > 3$, are ruled out.

This does of course not rule out particle graphs with more than three free edges. On the contrary, we know, owing to the pole conjecture, that corresponding to channel graphs such as



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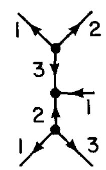
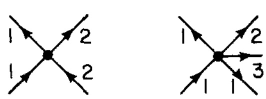


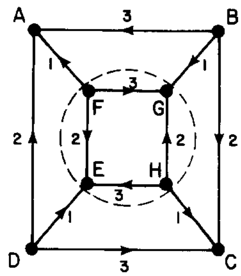
etc., there will be particles whose graphs have any number of free edges. But we have now seen that the particle graphs of these particles cannot be simple vertices such as



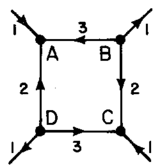
etc. We will soon see how these "exotic" particles are to be graphically represented. At any rate, since simple n vertices for $n > 3$ do not occur, and two-vertices are trivial, all process graphs are necessarily *cubic*.

9.2.4. Cubic Reducible Graphs: The Set of Legal Process Graphs. We have established that process graphs are necessarily cubic, three colored, and vertex oriented, and consist of baryon vertices, antibaryon vertices, and trivial vertices. But can *every* graph with these characteristics be a process graph of some ordered process, or are there further restrictions on the possible forms of process graphs?

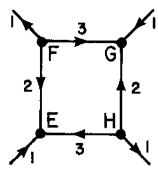
A simple example shows us that the latter is the case. Regard the "cube graphs"



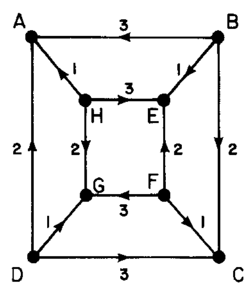
bisected as shown; this yields the two channel graphs



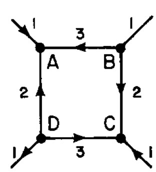
and



We see that there are two ways of resewing these channel graphs, one yielding the original process graph, and the other

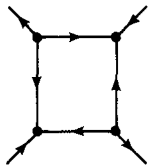


This corresponds to the existence of a nontrivial automorphism on the channel graph

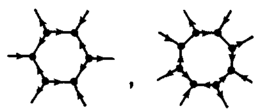


mapping A into C and vice versa, and B into D and vice versa, and the edges correspondingly.

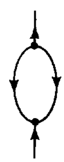
Since this violates unique resewability, such process graphs are ruled out; indeed every process graph containing



with any color assignment of edges, as a channel graph is ruled out. And for similar reasons all channel graphs with such "loops," such as



etc., are also rules out (those with an odd number of vertices are ruled out anyway by bipartiteness). The only exception is the two-vertex loop

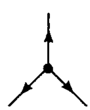


that we have already recognized as a legitimate channel graph, as it poses no problems with respect to unique resewing.

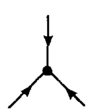
So as at least some cubic bipartite, three-colored graphs are forbidden, the question poses itself: Which process graphs are allowed?

To answer this question, we first use the general properties listed at the beginning of this section to establish a *minimal* set of process graphs, which *have* to occur: the cubic reducible graphs. And then we show that this is the complete set: there are no other process graphs than these.

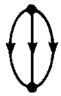
Starting from the legal channel graphs (particle graphs)



and

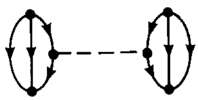


we can certainly (by Section 9.1.2) form the process graph

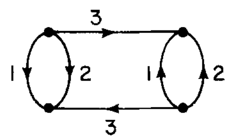


and (by Section 9.1.4) insert any number of trivial vertices on the edges, e.g.,

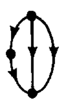
By Section 9.1.3 we know that we can compose two such process graphs, e.g., by one of their trivial vertices



to form the process graph



which is thus also legal. Again we can insert a trivial vertex on any edge and compose this graph with the graph



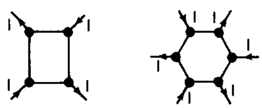
In this way we can insert "necklaces"

repeatedly on any edge, and so (see Section 8) *any* cubic reducible process graph is legal. And every bisection of a cubic reducible process graph yields a legal channel graph.

In Section 8 we examined the properties of vertex-oriented, three-colored cubic reducible graphs, and saw in particular that they satisfy the criterion of unique resewability. If this had not been the case, our whole general ordered SM approach would have failed, since the minimal class of process graphs would have failed to satisfy a basic requirement. As it is, our theory passes an important consistency test.

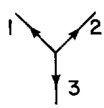
Specifically, we saw in Section 8 that any bisection of a three-colored vertex-oriented cubic reducible process graph yields two channel graphs with conjugate skeletons (in the bra–ket formalism, where the "out" channel graph is charge conjugated, the skeletons are identical). Conversely, any two such channel graphs can be sewn together to a cubic reducible graph if and only if they have conjugate skeletons, and then only in one unique way. This implies that all ordered channels from the same sector have the same skeleton; *the skeleton characterizes an ordered sector.* As we saw, skeletons are oriented and three-colored. In Section 10 we will additionally attach flavor labels to the free edges of skeletons. Such flavor-labeled skeletons fully characterize a sector, and hence also particles, in the sense that there is a one-to-one relation between ordered sectors and flavor-labeled skeletons. Without the flavor labels there is a set of sectors corresponding to each skeleton.

Thus reducible graphs certainly fulfill all the requirements of legal process graphs; the question remains whether there is a bigger class of graphs that also does. The answer is: no. Any three-colored, vertex-oriented cubic graph that is not reducible contains channel graphs of the kind

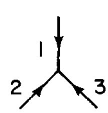


etc., which we have seen to be forbidden; this is connected with the fact that reducible graphs are just the ones with the property that all channel graphs have unique spanning trees.

To summarize: ordered processes can have only process graphs that are three-colored, vertex oriented, cubic reducible process graphs; and conversely every such graph corresponds to a possible process graph of some ordered process. So far we have only regarded processes with particles corresponding to simple vertices: baryons



antibaryons



and mesons

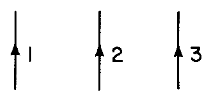
Now we complete the foundations of the theory by introducing the full hadronic spectrum generated by self-consistency requirements.

9.3. Some Other Properties

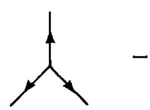
9.3.1. The Full Hadronic Spectrum. According to the pole conjecture every ordered channel communicates with at least one pole (particle). Any two ordered channels from the same sector communicate with the same set of poles, which establishes the correspondence between sectors and particles noted at the beginning of this chapter. Since sectors are characterized by skeletons, the same is true for particles: to every particle corresponds a three-colored, vertex-oriented tree graph, the skeleton, which we call its *particle graph*, and conversely to every such tree graph corresponds a set of particles differing from one another only in mass, spin, and parity, that are said to form a *skeleton class* of particles.

We list the simplest and most important skeleton classes of particles:

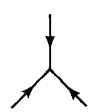
nonexotic mesons, with skeletons:



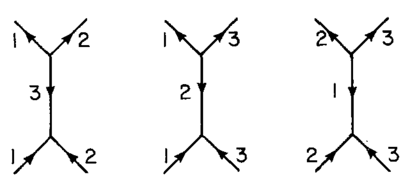
nonexotic baryons, with skeleton:



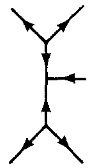
and antibaryons:



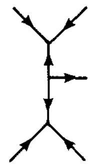
baryonium (exotic mesons):



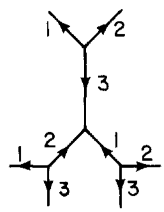
exotic baryons:



and antibaryons:



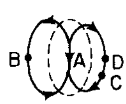
each in three colorings,
one "deuteron":



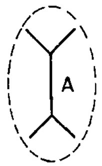
etc.

The spectrum represented by all such cubic tree graphs is complete; no other particles are compatible with the structure of the theory. Note the *zero-triality* nature of the spectrum: the number of edges directed away from the particle graph minus the number directed towards it is always a multiple of 3. With the identification of directed edges and "*quarks*" this feature will, in conjunction with the introduction of edge flavor, lead to the usual quarklike hadron spectrum predicted by quark theories. But in addition to the nonexotic mesons and baryons we obtain a well-defined set of exotic particles, the simplest of which, baryonium, may already have been found, and which will provide one of the crucial tests of this theory.

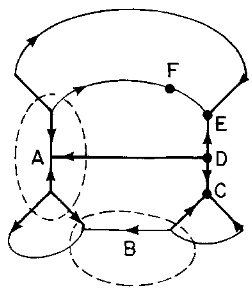
All these particle graphs can be combined in the usual way (colors and orientation of the free edges joined together have to match) to process graphs. We only demand that the resulting graph be reducible, and co-connected. The latter demand was automatically fulfilled as long as we were dealing only with particles corresponding to simple two-vertices and three-vertices: the removal of any such vertex still leaves a reducible graph connected. With the inclusion of exotic particles this is no longer guaranteed, so we have to mention it explicitly. For example, the process graph



connecting a baryonium A with several nonexotic mesons, is illegal, since removal of the baryonium, indicated here by



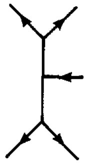
leaves the graph disconnected. This explains why baryonium cannot, at the ordered level, decay into a purely mesonic channel. This example also demonstrates the way we graphically indicate exotic particles in the context of a process graph: we surround the particle graph with a dotted line so that it is apparent that this represents one particle, and not a collection of baryons and antibaryons. Another example of a process graph containing exotic particles is



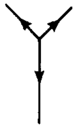
9.3.2. Baryon Number Conservation. The number of baryon vertices minus the number of antibaryon vertices in a particle graph or channel graph is called the *baryon number* of that particle or channel. It is apparent that the baryon number is additive: the baryon number of a channel is the sum of the baryon numbers of the component particles. And since there is always an equal number of baryon and antibaryon vertices in a reducible cubic graph (see Section 2), the baryon number of the "in" channel is equal and opposite to that of the "out" channel; in the bra–ket notation the two numbers are equal. Thus baryon number is conserved.

9.3.3. The Generalized OZI Rules. In its most general form the OZI rule says that if, given a set of particles, no legal process graph containing these particles can be constructed, then any ordered amplitude with these particles as external particles is zero, a truly trivial statement. Since the planar approximation to the SM is then also zero, and to the extent that it is a good approximation to the physical SM, physical amplitudes with such a set of particles would then be suppressed.

For example, we saw that baryonium cannot decay into a purely mesonic channel, because one cannot construct a channel with a baryonium skeleton from nonexotic mesons. Similarly, a particle from the skeleton class

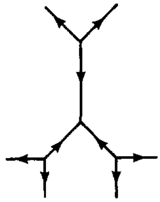


cannot decay into a baryon



and mesons

even though baryon number conservation would allow it. Another example is provided by

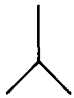


which cannot decay into two baryons, a fact that makes its identification with the physical deuteron dubious; instead, it may, for example, decay into three baryons and an antibaryon. For the sequentially ordered SM there was no analog of these selection rules, since there all particles have the same skeleton

(if we ignore flavor).

With the introduction of flavor, the OZI rule implies a whole class of selection rules analogous to those of Part One, as will be seen in Section 10. These selection rules often rule out processes at the ordered level that would be allowed according to quantum number conservation alone.

9.3.4. Why Are There Three Colors? We saw earlier that the existence of more or less than three colors is inconsistent with the existence of three-vertex particles



so, postulating the existence of such particles, and the general requirements of SMT, we were able to arrive uniquely at the theory described above.

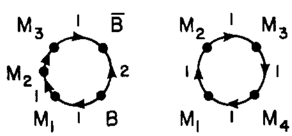
However, if we relax the demand that three-vertex particles exist, then there appears to be no a priori reason why theories analogous to this one, but containing a different number of colors, should not be self-consistent.

For example, a two-color theory would contain "baryons"

antibaryons

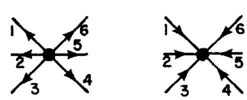
and mesons

Any process graph constructable from these particles would be legal, e.g.,



etc.

Theories with more than three colors have to be restricted to reducible graphs, as in the three-color theory; the process graphs of an n-color theory consist of "baryonic" and "antibaryonic" n-vertices like



and trivial vertices

The spectra are infinite, consisting of all possible tree graphs constructable from n-vertices. However, there are certain inelegancies in these theories, connected with the special role of the three vertex in graph theory. Since we have not completely worked out the arguments, we do not elaborate further here.

J. P. Sursock, using a related formalism, makes an interesting argument in favor of three colors, also based on the special graph-theoretical role of the three-vertex (Sursock, 1978).

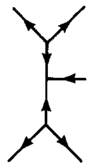
In summary, there exist theoretical arguments in favor of three colors but a rigorous proof has yet to be developed. The important point is that within this framework the prospects for such a proof look promising.

9.3.5. Are Baryons More Fundamental Than Other Particles? Considering the fact that all particle graphs are composed of baryonic and antibaryonic vertices, which themselves form the particle graphs of baryons and antibaryons, one might be tempted to think that baryons and antibaryons are the fundamental constituents out of which all other particles are made. Even mesons can be constructed out of baryonic and antibaryonic vertices, as the graph

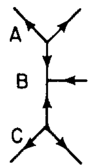
shows (although this graph is not a tree graph, unlike all other particle graphs constructed out of baryonic and antibaryonic vertices). This would contradict the SMT principle of "nuclear democracy," according to which no hadron is more fundamental than any other hadron, and no "fundamental constituents" or elementary hadrons exist out of which all others consist; instead, to the extent that the concept of "consisting of" makes any sense at all at this level, one could say that each hadron consists of all other hadrons. It can be shown that "consisting of" is a concept that strictly speaking only makes sense in the nonrelativistic limit, when the "binding energy" is small compared to the rest energy.

It is easy to see that the interpretation of baryons and antibaryons as fundamental constituents is wrong, and that in ordered SMT nuclear democracy holds with a vengeance, since the new aspect of order allows a very graphic description of the seemingly paradoxical statement that "every hadron is composed of all other hadrons."

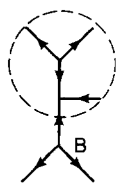
To see this, we remember the intimate correspondence between particles and sectors. Any particle regarded as a one-particle channel, communicates with all other channels of that sector, and can be considered to "consist" of the particles of any of these channels. For example, a particle with skeleton



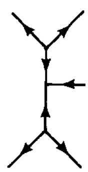
"consists" of two baryons and an antibaryon (as the graph

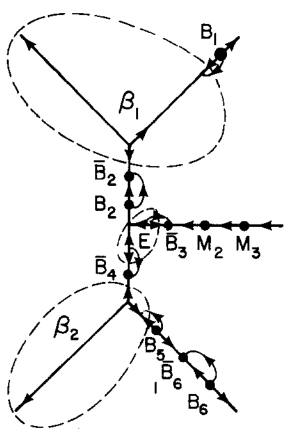


shows) or a baryonium and a baryon

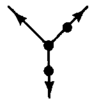


or of three mesons, four baryons, four antibaryons, two baryonium, and a

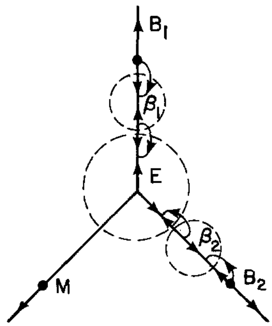




Similarly, a baryon "consists" of a baryon and two mesons

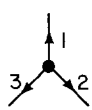


or of two baryoniums, an exotic antibaryon, a meson and two baryons

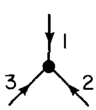


We see from these examples that baryons are no different from any other hadrons; all hadrons can indeed be regarded as composites of other hadrons. Particles, even those representable by simple vertices, are not irreducible entities, but categories of order. Order amongst what? One might give a circular answer so typical of SMT and say: order amongst the other particles it reacts with.

But we suspect that there is a deeper aspect to this of which we have had only glimpses and intimations. What do the vertices stand for; what do they represent? Our original idea that there was a one-to-one relation between vertices and particles, that the former represented the latter, has proven oversimplified (the statement is true only for mesons and baryons, but not for exotics) although there obviously is some connection between them. Rather, particles correspond to order categories (i.e., skeletons) between sets of fundamental vertices



and



And so how are we to understand vertices themselves? At the end of this work we will attempt some rather vague speculation about their meaning, but at this point we admit that their physical interpretation, as well as that of the order between them and the connection between ordered and physical SM, is still largely mysterious.

10. THE PROPERTIES OF THE ORDERED *S* MATRIX

In Section 9 we determined the form of order compatible with the principles of SMT, and developed the features novel to generalized order. Here we present the axiomatic structure of the theory, its resulting *S*-matrix properties, duality and quark-model properties, its symmetries, and the planar SM approximation to the physical SM. Thus this section, in the context of generalized order, covers the ground of Sections 4–6 of Part One. As will be seen, all the features of the sequentially ordered SM presented there will survive the transition to generalized order, albeit occasionally in somewhat modified form. This circumstance, coupled with the fact that the ordered SM passes stringent self-consistency conditions, in itself provides an encouraging sign that we are on the right track, quite aside from the empirical predictions the theory generates.

As the axioms, the deductions from them, and even the manner of deduction is very analogous to those of the sequential SM, we can afford to be very brief here, going into more detail only when nontrivial differences between the sequential and general case arise.

10.1. The Axioms of Ordered *S*-Matrix Theory

10.1.1. Order. Ordered particles are represented as three-colored, vertex-oriented cubic tree graphs; these particles can combine to ordered processes in any way such that the resulting process graph is a three-colored, vertex-oriented, cubic, reducible, coconnected graph. Any bisection of such a legal process graph yields a legal channel graph. To every ordered process corresponds an ordered amplitude (*T* matrix) expressing its probability in the usual way. (For details see Section 9.)

10.1.2. Lorentz Invariance. The ordered amplitudes are Poincaré invariant, and thus conform to the principle of special relativity.

Again, since Poincaré transformations do not affect particle order, but only the values of momenta and helicities, nothing need be added to what was said in Part One.

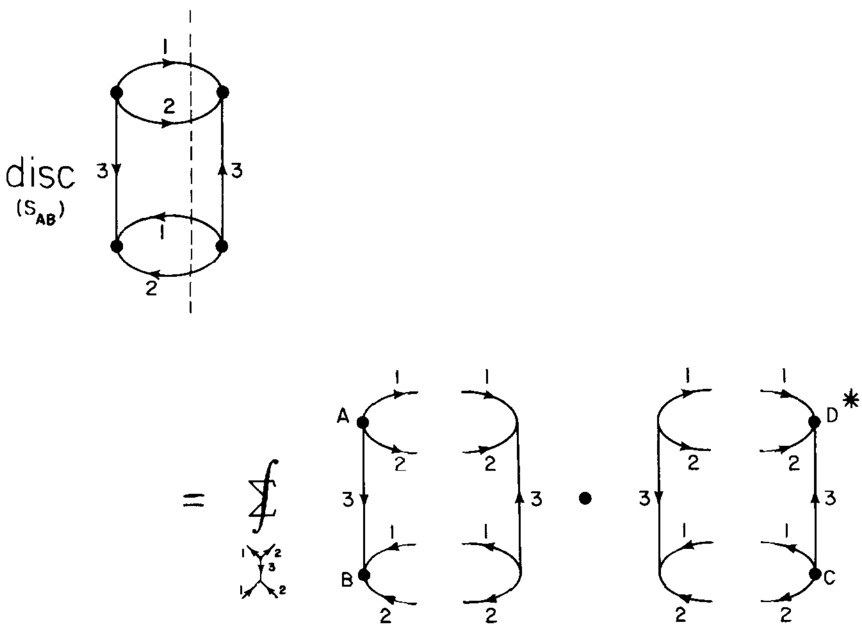
10.1.3. Cluster Decomposition. In order to express unitarity, we introduce an ordered *S* matrix, defined in terms of the above ordered amplitudes (connected parts, or T matrix) by means of an appropriate cluster decomposition equation; it is this ordered SM that is postulated to be unitary (see next axiom, Section 10.1.4).

In order to be an acceptable candidate for a cluster decomposition equation, it has to fulfill the following requirements:

(1) It has to be a generalization of the cluster decomposition of the sequential SM; i.e., when we apply it to an ordered SM containing only nonexotic mesons, it has to reduce to the cluster decomposition of Part One.

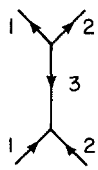
(2) It has to convert an independence postulate for the ordered SM into a corresponding statement about ordered connected parts (namely, that if any particle or set of particles of the ordered SM is translated to infinity, then the amplitude tends to zero). This is exactly as in Part One.

(3) When the unitarity equations for the ordered SM are converted into discontinuity equations for the ordered amplitudes by means of the cluster decomposition, the resulting discontinuity equations have the property that when the intermediate channels of the unitarity products are erased and the resulting channel graphs sewed together, one should obtain the process graph of the ordered amplitude whose discontinuity is being expressed. For example, the normal threshold discontinuity equation of the four-particle ordered amplitude obtained from unitarity and cluster decomposition must be of the form



where the symbol

includes a sum over all intermediate channels with the skeleton



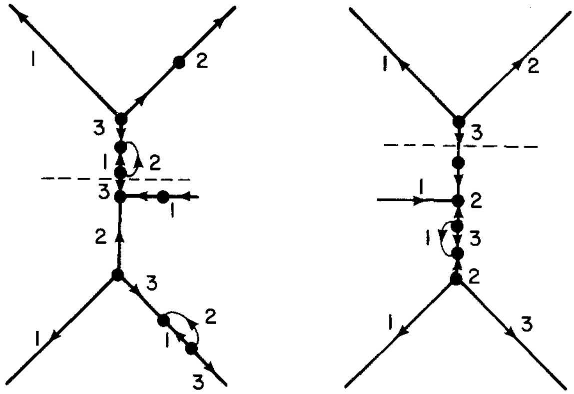
(4) It has to be consistent with crossing. By this we mean the following: Because of crossing (which will be shown to be valid), a discontinuity equation like the above example should be independent of the channel (i.e., of which particles are "in" and which are "out"); on the other hand, the derivation of such crossing-related discontinuity equations is different for each channel: i.e., we start out from a different unitarity equation and employ a different cluster decomposition for each different channel. So the fact that we have to obtain the same discontinuity equation every time imposes a stringent consistency condition on the theory; that there should be a cluster decomposition that achieves this at all is far from trivial.

(5) A similar consistency condition is imposed by the fact that, say, the above four-particle discontinuity equation obtained as the connected part of the four-particle unitarity equation must coincide with the corresponding discontinuity equation obtained as the disconnected part of, say, the six-particle unitarity equation.

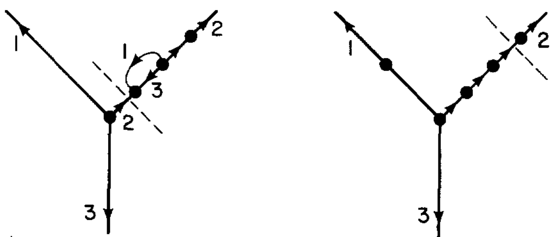
We have found only one cluster decomposition that satisfies all these conditions; that it exists at all, given all these restrictions, suggests again that there is some validity in the approach. We illustrate, in Section 10.1.4, with one nontrivial example, how the proposed cluster decomposition passes these tests.

We now describe the cluster decomposition. Firstly, if the initial and final channels of an ordered transition SM element do not have conjugate skeletons, then that element is zero. So let us assume conjugate skeletons.

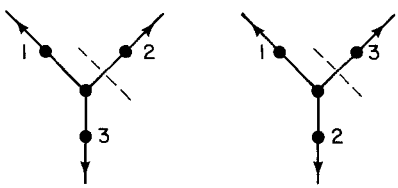
A cut separating the initial channel graph (and its skeletons) into two parts is called *corresponding* to a cut separating the final channel graph into two parts, if each of the pieces has a skeleton conjugate to that of its partner in the cochannel. For example,



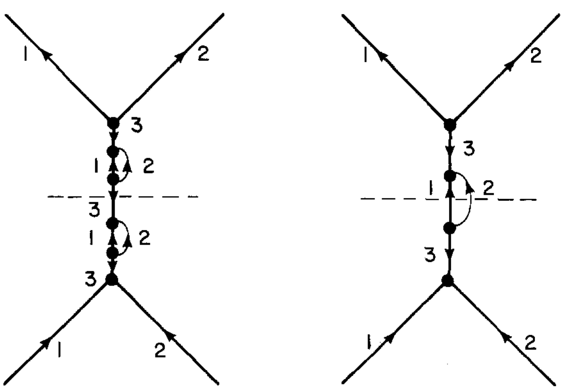
indicates corresponding cuts in the two channel graphs, as does



whereas the cuts

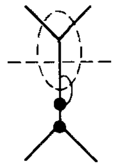


and also



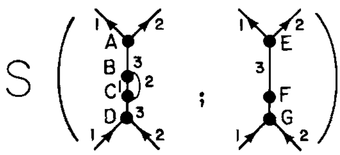
are not corresponding.

Needless to say, a cut should never pass through a particle, e.g.,

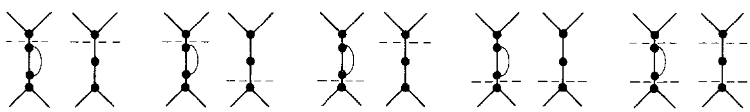


is not a legal cut at all, because it cuts through a particle graph (of a baryonium).

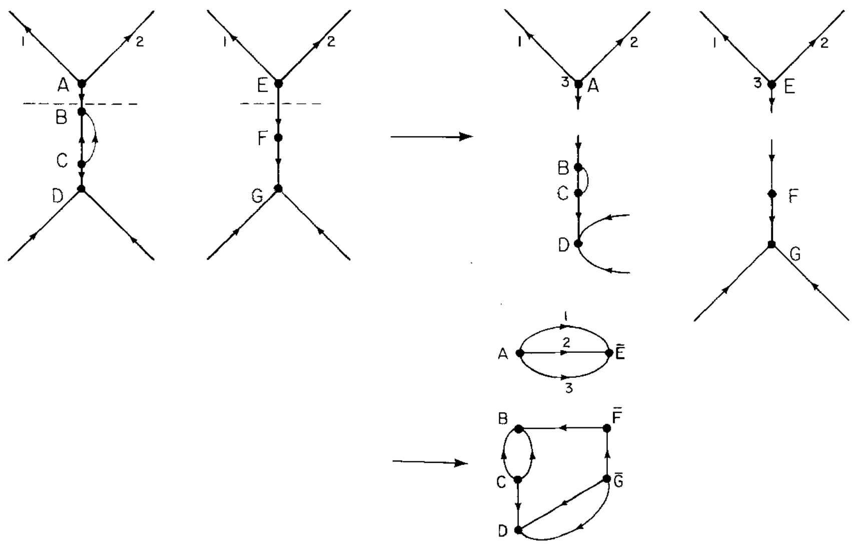
Given an ordered SM element, e.g.,



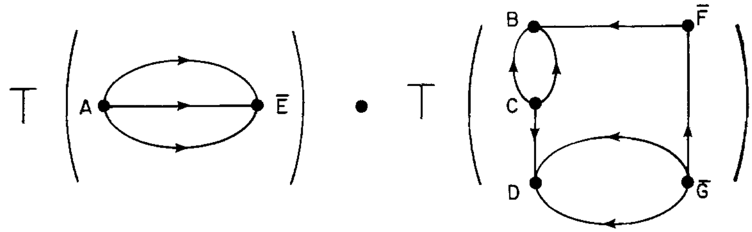
Then the cluster decomposition expresses this element as a sum of products of ordered connected parts. Each such term is obtained by cutting initial and final channels by a series of corresponding cuts and then resewing *corresponding* initial and final pieces to process graphs. In the above example the possible corresponding cuts are



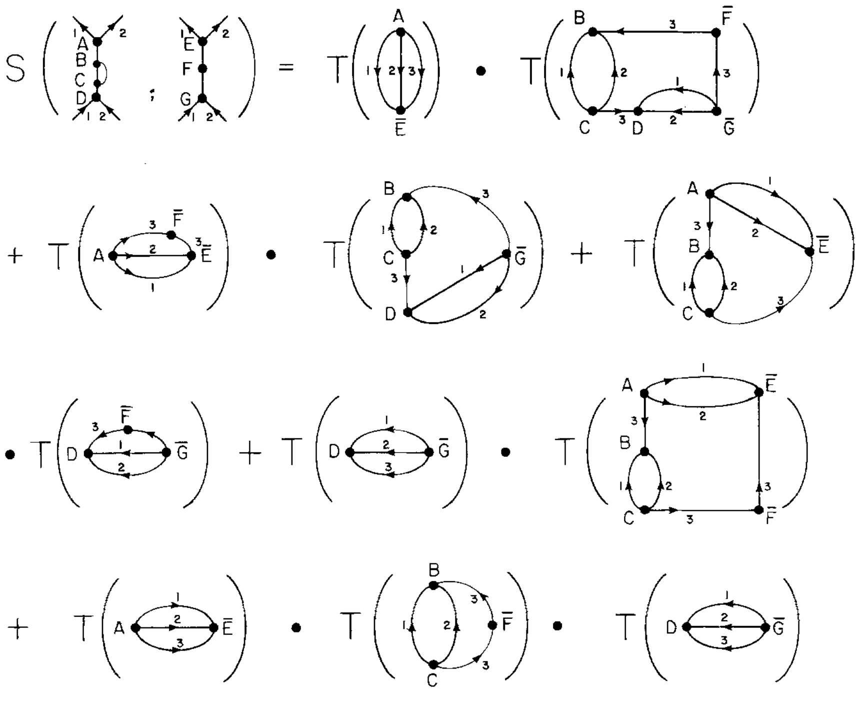
Let us regard the first pair of corresponding cuts as an example:



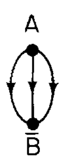
which leads to the cluster decomposition term



and similarly with all other pairs of corresponding cuts. This leads to the cluster-decomposition equation:

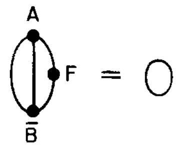


We often omit the $T(\ )$, and let the process graph stand for the ordered amplitude. As usual,



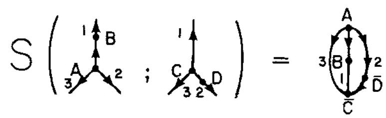
$$= \delta_{AB} = \delta_{t_A t_B} \cdot 2E_A \delta^3(\underline{p_A} - \underline{p_B}) \cdot \delta_{\mu_A \mu_B}$$

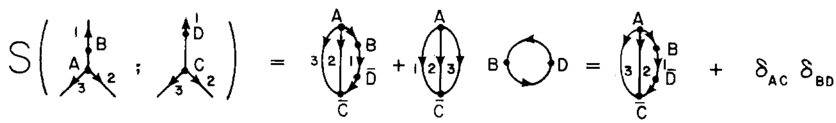
and

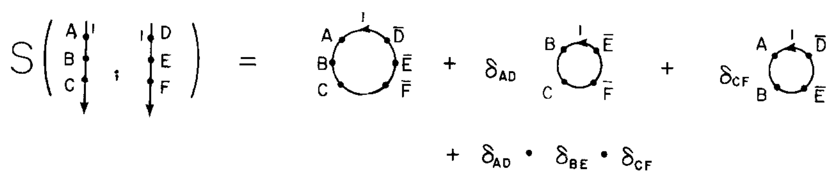


(stability).

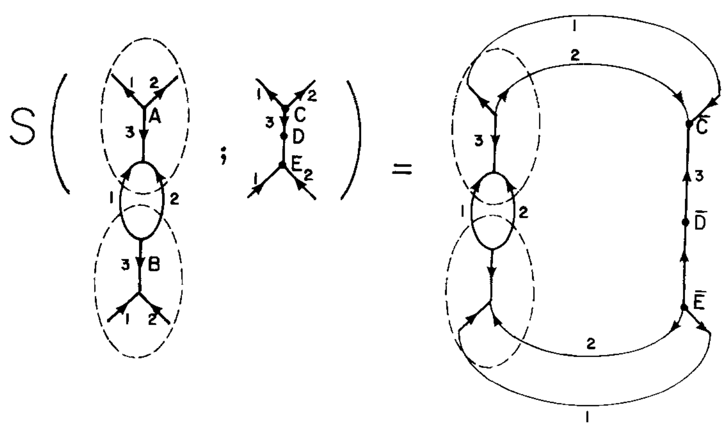
We give several more examples to establish the principle of cluster decomposition firmly in the reader's mind, since it is very cumbersome to express precisely in general terms:





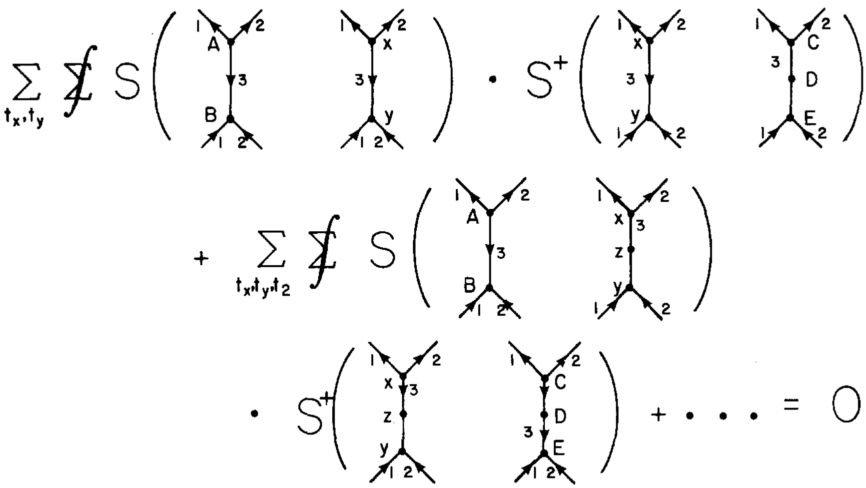


thus for sequential order, we get the same cluster decomposition as in Part One, as we should:



10.1.4. Ordered Unitarity. Having defined the ordered S matrix by means of the cluster decomposition we now postulate its unitarity: $\sum\!\!\!\!\!\!\int_n S_{in}S^+_{nf} = \delta_{if}$. As in the case of the sequential SM, the states i, n, and f are to be regarded as ordered states, in an ordered Hilbert space; it is there that the difference between ordered and physical unitarity lies hidden.

For example, we have

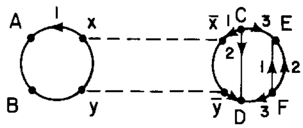


where $\sum\!\!\!\!\!\!\int$ stands for the usual phase-space integral and helicity sum, and $\sum_{t_x, t_y}$ stands for the sum over all particle types t_x, t_y, that can be inserted at that location to form the given skeleton. At any fixed energy the number of such terms is finite owing to the nonzero particle masses and the absence of accumulation points in mass.

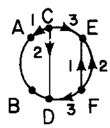
10.1.5. Macrocausality. Macrocausality is formulated just as for the sequential ordered SM, and with the same consequences: to every ordered amplitude corresponds a set of (ordered) Landau diagrams, representing the possible multiple scattering modes of that ordered process. Each such Landau diagram, in the usual way, determines by means of the associated Landau equations a positive-α Landau surface in momentum space, the set of all points in p space where that multiple scattering process is kinematically possible. And the postulate of macrocausality can then be shown to yield, in the usual manner, the analyticity of an ordered amplitude everywhere in the physical region (and a neighborhood) except its positive-α Landau surfaces; also obtained are the $+i\epsilon$ rules that tell us how to analytically continue the amplitudes around these singularities. All this is completely analogous to the sequentially ordered case and requires no further comment.

What does have to be explained here is which Landau diagrams correspond to a given ordered amplitude. As was the case for the simpler sequential order, a Landau diagram corresponds to a given ordered amplitude if not only its set of external particles is the same, but also the *global order* of the Landau diagram is identical with the order of the given amplitude. So what is the global order of a composite (multiple scattering) process, and hence of a Landau diagram?

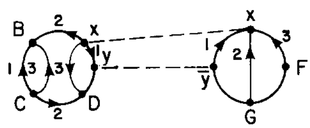
For the case of double scattering we have already described the answer in Section 9.1.3 (composability). If the internal particles in both subprocesses form an ordered channel (i.e., correspond to a bisection), and these two ordered (internal) channels are conjugate, then there is a global order, and it is obtained by erasing the internal channels in both subprocesses and resewing the remaining external channels (composition). We show some examples:



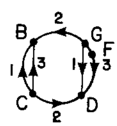
has a global order:



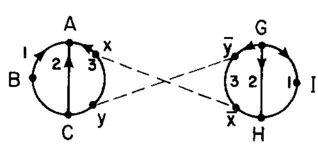
Likewise



has a global order

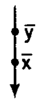


But

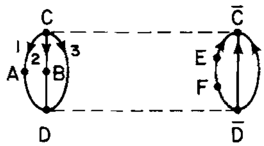


has no global order, because the internal channels

and

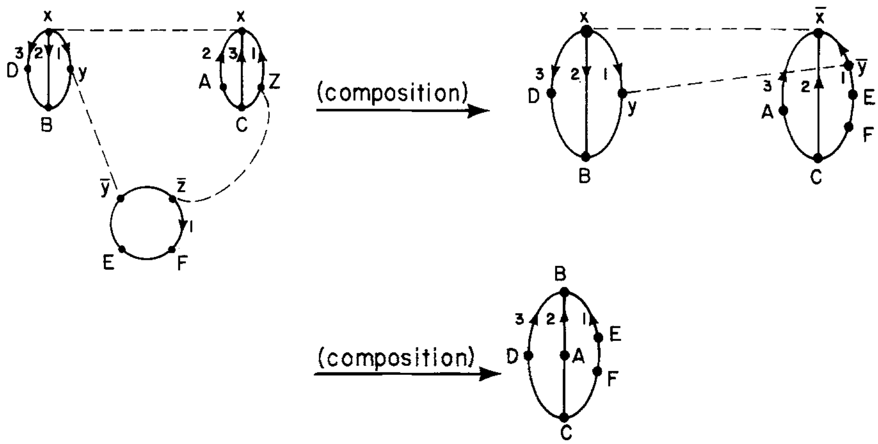


are not conjugate. And



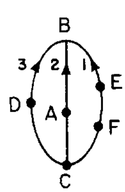
has no global order because the particles C and D do not form an ordered channel (in the left subprocess graph).

For the case of multiple scattering with more than two subprocesses, there is a global order if the pairwise composition can be successively carried out in any particular order of succession; if this is the case, then it can be carried out in every order of succession and yields the same process graph every time; this process graph represents the global order of the Landau diagram. Again this is as in Part One; e.g.,

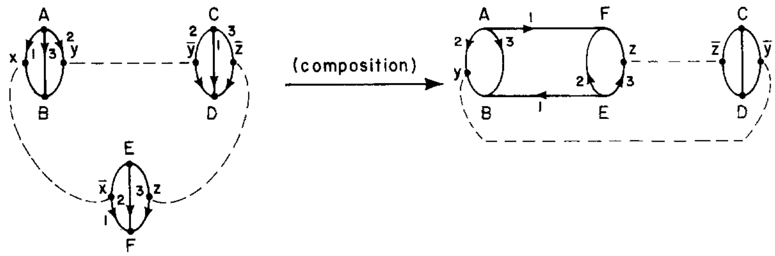


(the global order).

Therefore this is indeed a Landau diagram of the ordered amplitude



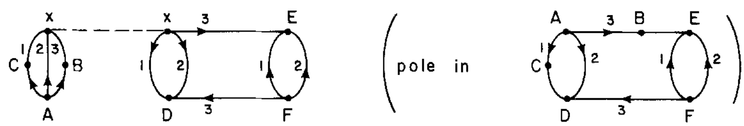
On the other hand, the Landau diagram



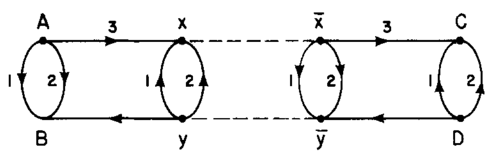
which latter, as we know, has no global order (since the internal particles do not form an ordered channel), and so the given multiple scattering diagram is not a Landau diagram of any ordered amplitude.

This definition of global order is further confirmed when we study connected-part unitarity and the resulting discontinuity relations for ordered amplitudes: the discontinuity of an ordered amplitude with process graph G around any Landau singularity is given by a multiple unitarity product of ordered amplitudes with global order G, as in the case of the sequential SM, and as it should be for consistency.

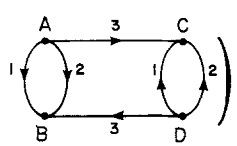
The most important singularities are the normal-threshold and pole singularities; they correspond to Landau diagrams consisting of two subprocesses, such as



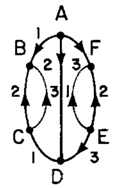
or



(two-particle normal threshold in



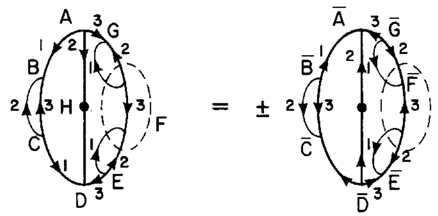
For these singularities the singularity structure of ordered amplitudes is particularly simple to formulate: *normal thresholds and poles occur only in* *channel variables corresponding to ordered channels, i.e., to bisections.* For example, the amplitude



has poles and normal thresholds in S_{AB}, S_{BC}, S_{CD}, S_{DE}, S_{EF}, S_{FD}, S_{ABC}, S_{BCD}, S_{CDE} and no others, in particular not in, e.g., S_{AD}.

Hence, in general, the singularity structure of ordered amplitudes is much simpler than that of physical amplitudes with important consequences for dispersion relations, Regge structure, and duality.

10.1.6. *C* Symmetry and Color Symmetry. We postulate the charge-conjugation symmetry of the ordered SM; as we have seen, C operates on an ordered state by converting each particle into its antiparticle, inverting the orientation of every edge, and multiplying by a phase factor ± 1 for every particle involved, which is specific for each particle. Thus, e.g.,

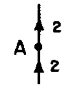


We now similarly postulate a *color permutation symmetry.* A color permutation on a graph, such as P_{12}, changes every edge of color 1 into one of color 2, and color 2 into color 1. For the three colors we are dealing with there are $3! = 6$ such color permutation operations that form the symmetric group of order 3.

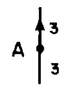
The physical postulate of color symmetry implies the following statements:

For every ordered particle A and every color permutation P there is a well-defined particle A_P of the same mass, spin, and parity, and with a particle graph obtained from the particle graph of A by application of P to it. For example, for a meson A with the particle graph

the application of P_{12} yields another meson $A_{P_{12}} \equiv A'$ with the particle graph

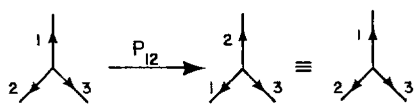


and P_{13} yields $A_{P_{13}} \equiv A''$ with the particle graph

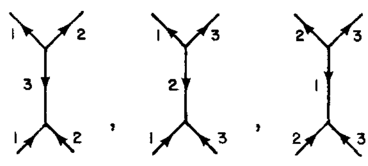


Both A' and A'' have the same mass, spin, and parity as A but are distinct. None of the other color permutations yield anything new. Thus mesons (on the ordered level) occur in color triplets.

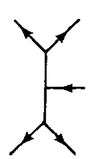
For baryons, as long as we are dealing with one flavor only, a color permutation leaves the particle invariant, except for a phase factor, which can have the values ± 1, $e^{\pm 2\pi i/3}$. How do we know that a color P permutation leaves a baryon A invariant up to a phase factor? This is because P leaves the particle graph of A invariant; e.g.,



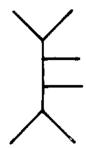
This means that A and $A_{P_{12}}$ are in the same sector. Since they also have the same mass, spin, and parity, we know from Part One that $A = A_{P_{12}}$; and similarly for any other baryon or antibaryon A and color permutation P. For similar reasons we knew that there were exactly three mesons in a color multiplet, and we know that there are three baryonium



three exotic baryons of the type

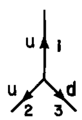


nine mesons of the type

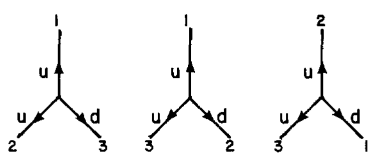


etc.

When, as we shall soon do, we flavor-label the free edges of particle graphs and sector skeletons, then nonexotic mesons still form color triplets, as before, but for the other particles the result is different, although the mode of reasoning stays basically the same. For example, for baryons with two edges with equal flavors, like



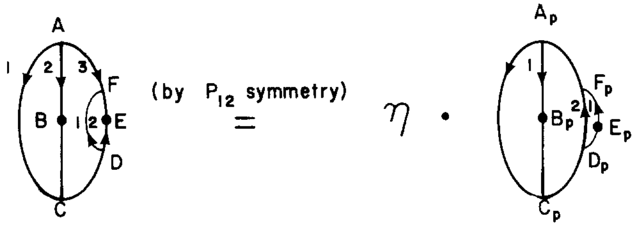
there are three particles in a color multiplet



And for baryons with three differently flavor-labeled edges there are six particles in a color multiplet.

We hope that when one will pass from the ordered to the physical SM by means of the topological expansion it will be shown that all the ordered particles in one color multiplet will correspond to one single physical particle; at this point this is only a conjecture, since the topological expansion and its consequences have not yet been fully worked out. But if this conjecture is borne out, then this theory may give a correct description of the full hadronic spectrum.

We can now state the postulate of color permutation symmetry: given an ordered amplitude and a color permutation, say P_{12}; if in its process graph we replace each particle A_i by the particle $A_{i_{P_{12}}}$, and replace edge color 1 by 2, and 2 by 1 everywhere in the graph, then the resulting ordered amplitude is numerically equal to the original amplitude, except for a phase factor with the possible values ± 1. For example,



where $\eta = \pm 1$.

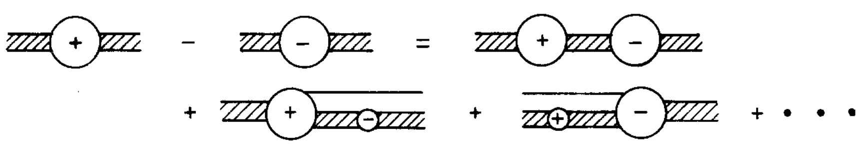
It is to be noted that there is a graph-theoretical symmetry of cubic reducible graphs with respect to reversal of edge orientation and to color permutation in the sense that any relation involving a set of such graphs (like, e.g., a unitarity equation) is preserved under these symmetry operations; this is a mathematical symmetry independent of any physics. Given the bootstrap definition of ordered amplitudes by means of sets of unitarity equations, this mathematical symmetry is a prerequisite for the postulated physical symmetry of the particles and amplitudes; but it is not a sufficient condition (as the example of broken flavor symmetry shows), so that we still need to *postulate* C and color symmetry, as we have done.

10.1.7. The Bootstrap Conjecture. The bootstrap conjecture holds that the combined demands of all the listed axioms so constrain the possible structures of the ordered SM that they actually only admit of one solution, namely, the one describing the real world.

In the case of ordered amplitudes the bootstrap conjecture takes on particular importance, since unlike physical amplitudes they are only extremely indirectly defined in terms of experiment (via the topological expansion). The bootstrap conjecture then at least has them defined in terms of their properties; an actual implementation of the bootstrap program is still likely to be formidable, though less so than for the physical SM.

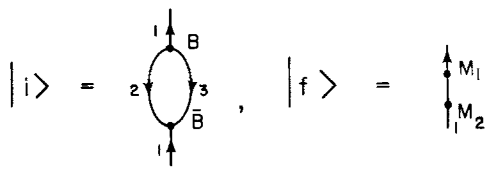
10.2. Derived Properties of the Ordered SM

10.2.1. Unitarity Relations for Connected Parts. If, in the unitarity equations for the ordered SM elements $\sum_n S_{in}S^+_{nf} = \delta_{if}$ we substitute the SM elements with their cluster decomposition into products of connected parts, then we obtain a set of equations expressing unitarity in terms of connected parts only, of the kind

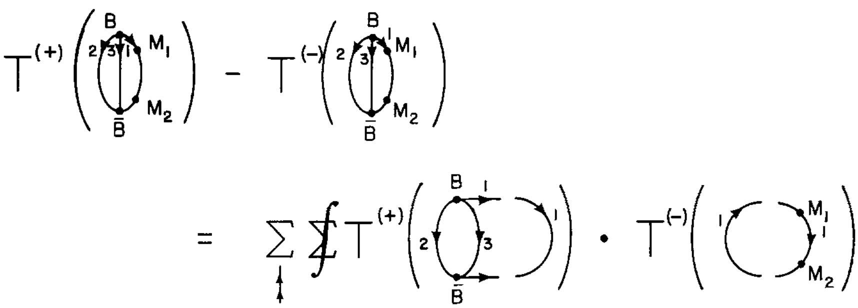


except that now the occurring amplitudes are all ordered. The global order of the unitarity products on the right-hand side is always identical to the order of the two left-hand terms, and conversely every such product contributes to the right-hand side. The various consistency conditions mentioned in Section 10.1.3 are all satisfied in the examples we have examined, but we have not proved consistency in general, nor do we consider the question of consistency as trivial.

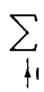
Examples: The unitarity equation for



yields the relation

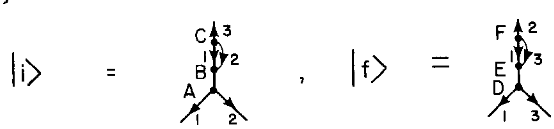


where

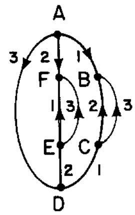


indicates a sum over all intermediate channels with the skeleton

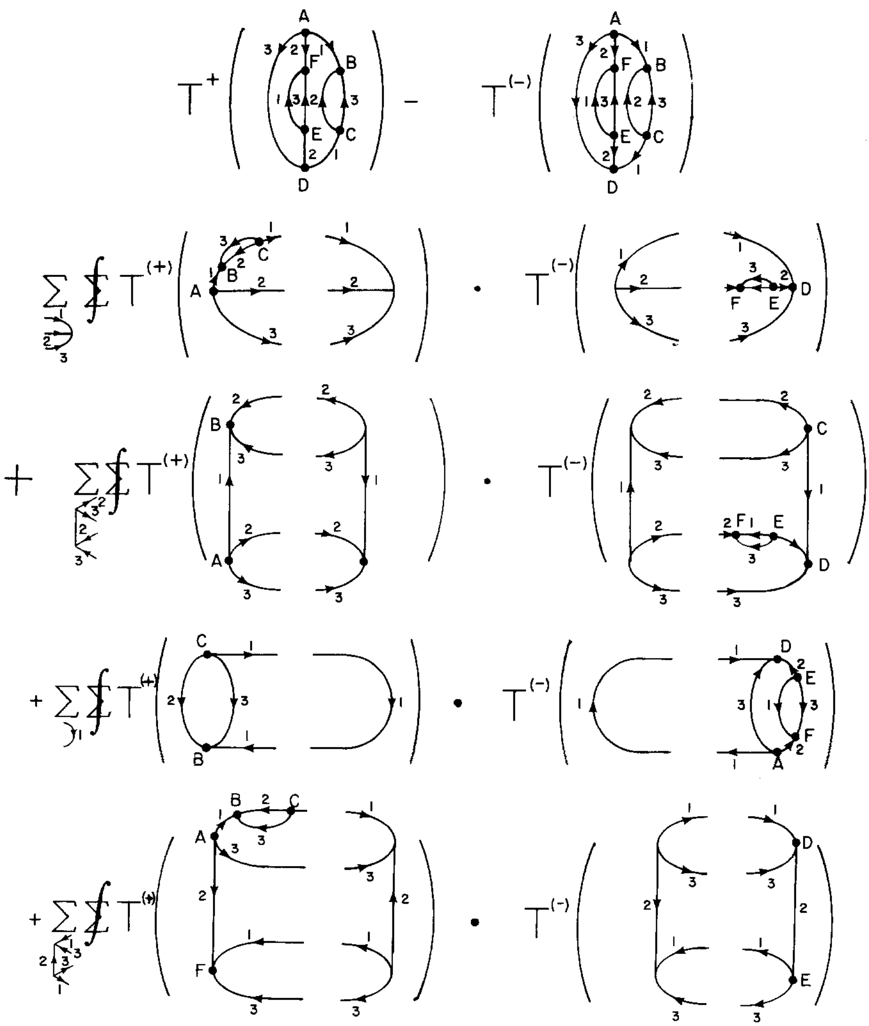
Similarly, if



(together forming the process graph)



then we obtain the relation



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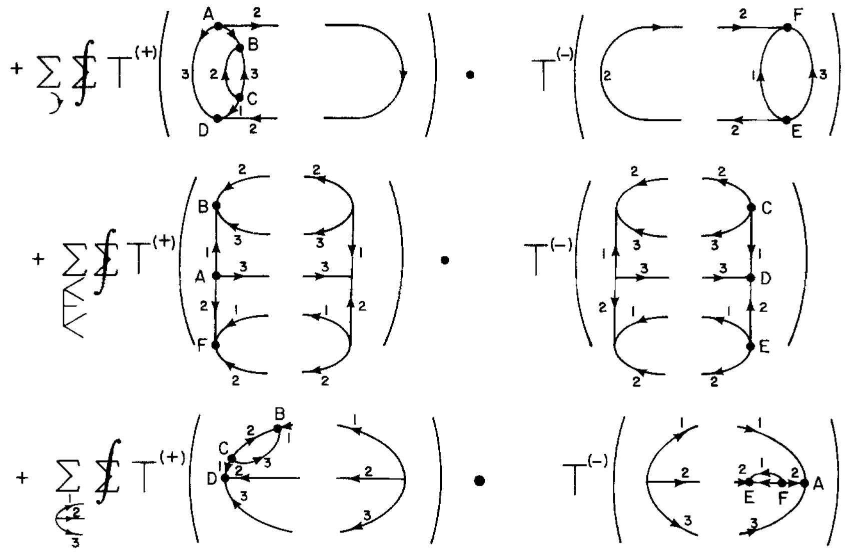
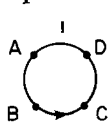


10.2.2. Discontinuity Equations. Starting from the above unitarity equations, one can again, by repeated use of unitarity and cluster decomposition, derive a set of discontinuity equations that express the discontinuity of an ordered amplitude around any of its Landau singularities in terms of appropriate unitarity products. In the simpler examples that we have checked (poles, normal thresholds, triangle singularities), the discontinuity formulas are formally identical with those of the physical and sequentially ordered SM. We believe that this is generally true, but have not proved it.

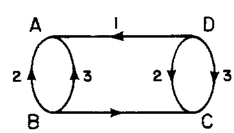
10.2.3. Pole Factorization. Pole factorization follows, as in Part One, from the pole discontinuity formula, and implies the pole–particle identity.

10.2.4. The Analytic Structure of Ordered Amplitudes and Duality. Macrocausality implies that ordered amplitudes are analytic in and around the physical region, and limits the locations of possible singularities to the positive-α Landau surfaces; on the other hand the discontinuity formulas imply that ordered amplitudes actually become singular on these surfaces. Thus the analytic structure of ordered amplitudes in the physical region is fully determined from first principles, as was the case for sequentially ordered amplitudes.

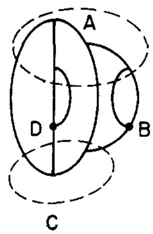
For four-particle amplitudes the consequences are particularly simple to express: they have normal-threshold cuts in only two of the three channel variables. For amplitudes like



or



this is obvious, but it is also true for more complicated amplitudes like the four-particle amplitude

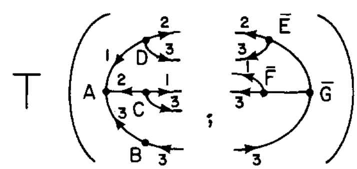


All these amplitudes have cuts in s_{AB}, s_{BC}, but not in s_{AC}. That the statement is generally true follows from the graph-theoretical fact that in a cubic, reducible coconnected four-particle graph only two of the three possible ways of partitioning the graph into two two-particle sets correspond to a bisection.

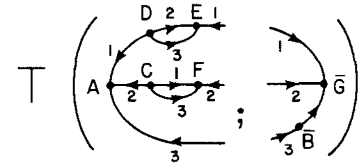
As a consequence, ordered amplitudes have the property that we called "duality" in Part One, implying exchange degeneracy, absence of Regge cuts, and the other properties mentioned there.

10.2.5. Crossing and TCP. The crossing property for ordered amplitudes can be proved from the basic postulates, plus a technical assumption about the singularity structure outside the physical region, quite analogously as for sequentially ordered amplitudes.

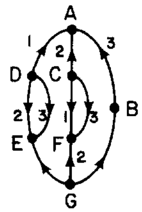
It states that two ordered transition amplitudes corresponding to the same process graph are related to one another by analytic continuation in the usual sense. For example,



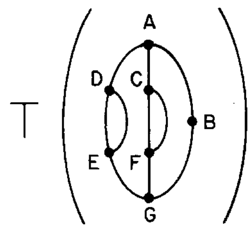
and



are related by crossing as is every other ordered transition amplitude obtained from the same process graph



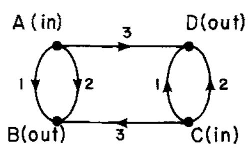
by bisection. They are all described by one and the same analytic function, which is thus a characteristic of the process graph, and not of the particular way in which the process graph is bisected into channel graphs. We designate this analytic function by



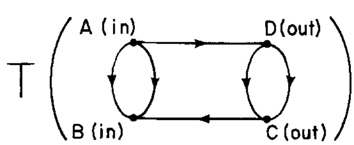
The $T(\ )$ may be omitted.

As was the case for sequentially ordered amplitudes, the significance of the concept of process graph, implying as it does a *joint* order of the "in"- and "out"-channel particles, rests upon the validity of crossing.

Again, we can regard the analytic continuation of ordered transition amplitudes to real-momentum regions corresponding to ordered processes which are not ordered transitions (i.e., where "in" and "out" particles do not each form a connected ordered channel) as *defining* the appropriate amplitudes; remember that up to this point we had introduced only ordered transition amplitudes. For example, the amplitude corresponding to the ordered process



is defined by the appropriate analytic continuation from the ordered transition amplitude



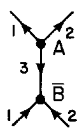
The TCP theorem follows, as for sequentially ordered amplitudes, essentially by repeated application of crossing.

10.2.6. Some Other Properties. The comments made in Part One, Section 3, about the validity of dispersion relations, Hermitian analyticity, extended unitarity, and the Froissart bound continue to hold in the context of generalized order, as does the remark about the pole conjecture.

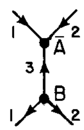
10.3. Flavor

Here again we can afford to be brief, since the ideas and arguments are closely analogous to those of Part One, Section 5. In summary, flavor again enters the picture via order selection rules that prohibit certain particles from being neighbors in a process graph; if there are any of these selection rules, then the free edges of particles and skeletons (sectors) can each be assigned a flavor label, and only free edges with the same flavor label (and of course color and orientation) are allowed to be sewed together in a nonzero ordered amplitude (OZI rule). We now describe this reasoning in more detail. For simplicity's sake we restrict ourselves to baryons here, but the procedure goes through with exotics too.

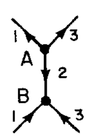
Regard any two baryons A and B. If



and hence also



are interacting channels, then we call A and B three-neighbors; similarly, if

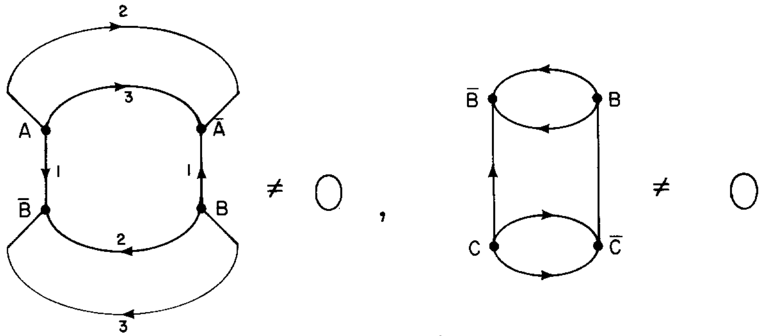


is interacting, then A and B are two-neighbors; and similarly for one-neighbors. Order of selection rules stating that an ordered channel is noninteracting can always be traced back to the existence of such noninteracting two-particle channels, as was the case for mesons.

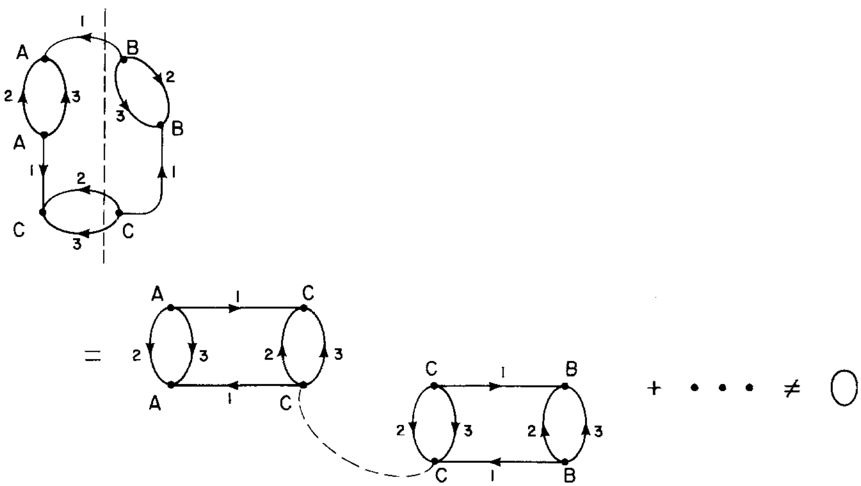
If A and B are one-neighbors, and B and C are one-neighbors, then A and C are also one-neighbors; i.e., being a one-neighbor is a transitive relation (the same is of course true for two-neighbors and three-neighbors). This is so because if

$$\left.\begin{matrix} A, & B \\ B, & C \end{matrix}\right\}$$

are one-neighbors, then



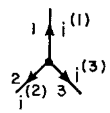
and so the discontinuity formula



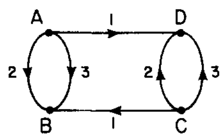
and so A and C are one-neighbors, as claimed.

Since the relation of being one-neighbors is also symmetric and reflexive, it is an equivalence relation; therefore the set of all baryons decomposes into equivalence classes in such a way that any two particles from the same class are one-neighbors, whereas any two particles from different classes are not. We label these classes with a discrete label $i^{(1)}$; analogously we define $i^{(2)}$ and $i^{(3)}$ to label the two-neighbor and three-neighbor equivalence classes.

These indices are called flavor labels. Obviously, every baryon belongs to a definite one-equivalence class $i^{(1)}$, a definite two-equivalence class $i^{(2)}$, and a definite three-equivalence class $i^{(3)}$; we say it is in the class $(i^{(1)}i^{(2)}i^{(3)})$, and denote this by

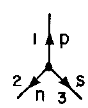


It is clear by construction that the two free edges of any two particles joined together in a nonzero process graph must have the same flavor; e.g., in

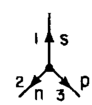


we must have $i_A^{(1)} = i_D^{(1)}$, $i_B^{(2)} = i_A^{(2)}$, $i_B^{(3)} = i_A^{(3)}$, etc. (*OZI* rule). As a result, all edges of a nonzero process graph can be uniquely flavor-labeled. With this achieved, the identification of process graphs with quark diagrams and of edges with quarks follows as in Part One, Section 5.

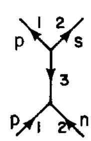
We again postulate that there are no selection rules besides order selection rules; thus an ordered amplitude is nonzero if and only if it satisfies the above OZI rule. And sectors are characterized by skeletons with their free edges flavor-labeled, as are therefore the particles themselves. For example,



characterizes a particular sector, distinct, e.g., from the sector



Similarly



characterizes a sector (note that inner edges are not flavor-labeled), etc.

A priori, there is no relation between the sets of indices $i^{(1)}$, $i^{(2)}$, $i^{(3)}$.

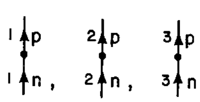
But owing to color symmetry, a one-to-one relation exists between these sets: to every flavor $i^{(1)}$ there is a corresponding flavor $i^{(2)}$ and a corresponding flavor $i^{(3)}$. That is why we may, e.g., talk about strangeness rather than about one-strangeness, two-strangeness, and three-strangeness. In other words, we can regard color and flavor separately: e.g., $i^{(2)} \equiv (i; 2)$: flavor i and color 2.

As for the sequentially ordered amplitudes given n flavors, we can define a set of $3n$ canonical additive conserved quantum numbers $Q^{i^{(\alpha)}}$ that indicates the net number of edges of color α and flavor i coming out of a particle or channel. Of physical importance are the n canonical quantum numbers

$$Q^{(i)} = \frac{Q^{(i^1)} + Q^{(i^2)} + Q^{(i^3)}}{3}$$

from which the color degree of freedom has been eliminated. Their importance results from the fact that when we pass via the topological expansion to the physical SM, then the only selection rules surviving are just those expressible as $Q^{(i)}$ conservation; thus the canonical $Q^{(i)}$'s are the only additive conserved quantum numbers at the physical level. Since the details of the topological expansion for the generalized ordered SM are still in the process of being worked out, the above statement remains a likely conjecture at this point.

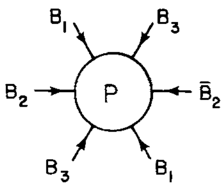
One usually uses a slightly different set of quantum numbers, replacing the set $Q^{(p)}, Q^{(n)}, Q^{(s)}, Q^{(c)}, Q^{(t)}, \ldots$ by the linearly related set $Q^{(\mathrm{el})}, Q^{(B)}, Q^{(s)}, Q^{(c)}, Q^{(t)}, \ldots$, where $Q^{(\mathrm{el})}$ is the electric charge mentioned in Part One and $Q^{(B)} = \sum_i Q^{(i)}$ is the baryon number. In terms of these quantum numbers the spectrum of particles obtained is the usual zero-triality quark-model spectrum. And when we add the postulate of $SU(n)$ flavor symmetry, we obtain the familiar quark-model multiplet structure. What still has to be better understood (on the basis of the topological expansion) is the way that color is eliminated and the multiplicity of particles reduced in the transition from the ordered spectrum to the physical spectrum of particles. For example, we would like to obtain only one physical pion ($p\bar{n}$) corresponding to the three ordered particles

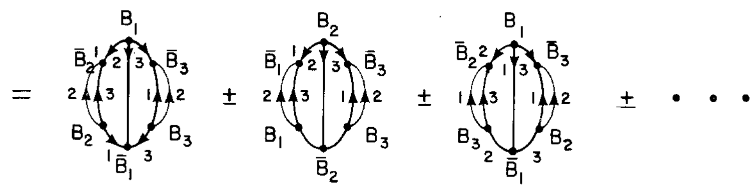


10.4. The Planar Approximation

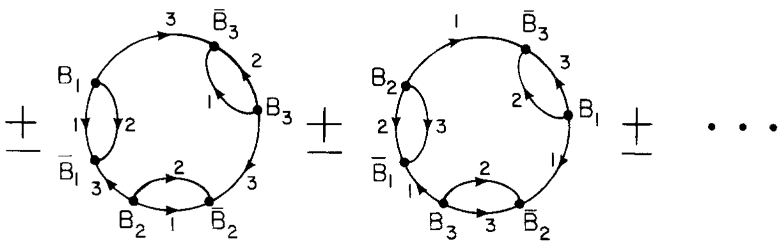
As was the case for the sequentially ordered amplitudes (see Part One, Section 6), when we wish to calculate the physical amplitudes, we have to eliminate ("average out") the particle order. This is done by means of a generalization of the topological expansion, the details of which are being worked out (Sursock, 1978). The lowest-order term of the topological expansion is the planar approximation, obtained by adding or, respectively, subtracting all ordered amplitudes with a given set of external particles. These planar connected parts, being unordered, can be directly compared with the physical connected parts.

Let us regard a simple example, a planar amplitude involving three baryons and three antibaryons. Then we have





... ± (all other terms of the type



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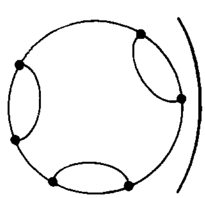
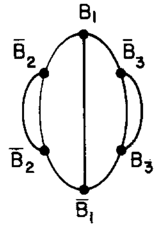
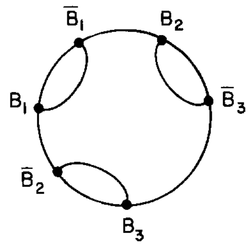


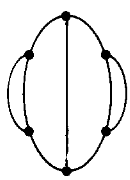
How is the relative sign of any two terms, say



and



determined? The rule that we propose here is the only one that we were able to find that yields the correct statistics and also determines the relative sign, in a nonarbitrary way, of ordered amplitudes with nonisomorphic process graphs, such as in the example above



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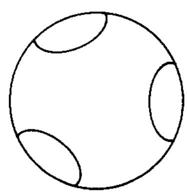


The rule is based on the "mate assignment" of the two terms in question. Thus in the first example above, B_1 and $\bar{B}_1$, B_2 and $\bar{B}_2$, and B_3 and $\bar{B}_3$ are mates, which we may denote by

$$\begin{pmatrix} B_1 & B_2 & B_3 \\ \bar{B}_1 & \bar{B}_2 & \bar{B}_3 \end{pmatrix}$$

whereas in the second example the mate assignment is

$$\begin{pmatrix} B_1 & B_2 & B_3 \\ \bar{B}_1 & \bar{B}_3 & \bar{B}_2 \end{pmatrix}$$

In this case the permutation relating these two mate assignments is odd (it involves an odd number of transpositions, namely, one: the transposition

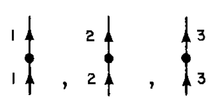
$\bar{B}_2 \leftrightarrow \bar{B}_3$); the rule then states that the relative sign of the two terms is negative; if the permutation had been even, the sign would have been positive.

With this rule governing the signs, we are guaranteed the correct statistics: Bose statistics for identical particles with an even number of free edges, and Fermi statistics for those with an odd number of free edges.

One might ask whether there are any theoretical reasons why particles with an odd number of free edges must be fermions, and with an even number of bosons (empirically this is of course true). As we have seen, every particle occurs as a pole in some purely baryonic channel; if the particle has an even (odd) number of free edges, then the number of baryons and antibaryons in the channel is necessarily even (odd). From this one can conclude that particles with an even number of free edges are necessarily bosons, whether baryons are fermions or bosons. And particles with an odd number of free edges are bosons if baryons are bosons, and fermions if baryons are fermions. Therefore the basis question to ask is why baryons are fermions, as we know empirically that they are. One way of answering this is by predicting the spin of a baryon. If we can show that baryons have half-integral spin, then the spin-and-statistics theorem shows that they must be fermions. We have not yet explored the whole problem of spin, and must therefore leave the question at that.

The general planar amplitudes we have defined here have the same properties that we found mesonic planar amplitudes to have in Part One, Section 6, and for the same reasons; thus little more need be said here. They also lack exact unitarity, but it is to be hoped and expected that there will be mechanisms in the generalized topological expansion, as there were in the mesonic one, that will suppress nonplanar terms and explain the approximate planarity found in nature. One argument is already apparent: since planar amplitudes obey pole factorization, which plays the role of unitarity with one-particle intermediate states, we can expect planar amplitudes to be approximately unitary to the extent that they are resonance-dominated. Also, $1/N$ arguments will suppress nonplanar terms.

Another important problem that has to be solved in the context of the topological expansion is the correspondence between ordered and physical particles. The presence of color renders this relationship more complicated than in the sequential case. For example, we expect that to the three ordered particles



related by color symmetry there corresponds one physical particle (meson). Only when this relationship will have been worked out will one be able to make completely specific predictions about the spectrum of hadrons and its multiplet structure.

11. CONCLUSION

11.1. What Has Been Achieved

By extracting the essence of the duality approach, we have arrived at the concept of sequential particle order; by combining it with the basic principles of S-matrix theory we have constructed a theory of the sequentially ordered S matrix that describes nonexotic mesons, and provides a theoretical foundation for the successful group of models known as DTU. Besides the usual S-matrix properties the theory predicts the duality and quark-model properties of amplitudes including the OZI rule, the nature of the mesonic spectrum and most of the other observed regularities of mesons. In particular, all the quark-model predictions regarding mesons other than those pertaining to spin and parity (which we have not yet incorporated into our scheme) are reproduced by the ordered SM approach. A totally different understanding of the origin of the quark structure of hadrons is thus gained: quarks as order relationships between particles rather than as constituent particles.

This theory is then extended to all hadrons by a generalization of the concept of order. This generalization is so restricted by consistency conditions as to be unique and yields a three-color zero-triality quarklike hadron spectrum including nonexotic mesons and baryons as well as a well-defined set of exotic hadrons including baryonium. All the general features of the sequentially ordered theory mentioned above are maintained. Thus, in principle, the extension of DTU to all hadrons appears to have been achieved.

Together with a topological expansion that expresses physical amplitudes in terms of ordered ones, the ordered S-matrix approach appears to offer the greatest hope yet of a successful theory of hadrons and strong interactions.

11.2. What Remains to be Done

It is our opinion that in the direction suggested by this paper there lies a vast new territory waiting to be systematically explored.

On a more immediate and technical level, a host of problems have to be solved. We just mention a few of them here.

For example, the proof that the theory requires exactly three colors needs to be tightened up. The connection between particle order and peripherality deserves examination, as does the origin of order selection rules (flavor) and internal symmetries, especially $SU(2)$.

By understanding the origin of order selection rules, possibly on the basis of topological structure, the nature and properties of flavor might be illuminated.

Consideration of spin and parity should be introduced, allowing their prediction as in the $SU(6)$ scheme, and also allowing us to understand the relative success of nonrelativistic potential quark models of charmonium. In which sense can one attribute spin 1/2 to the edges? Why are baryons necessarily fermions?

A systematic Regge theory of ordered amplitudes can be developed, leading (because of the simpler singularity structure) to a most probably considerably simplified Reggeon calculus. Are ordered amplitudes really, as conjectured, devoid of cuts and fixed poles in the j plane?

The relation between the ordered and the physical amplitudes and their respective spectrum of particles has to be better understood on the fundamental level. The generalized topological expansion has to be constructed, including the elimination of the color degree of freedom. The resulting detailed predictions of the spectrum of physical particles have to be worked out. How does "mixing" arise? Why do some nonstrange baryon trajectories occur without an exchange-degenerate partner? What is the multiplet structure of exotics? Their widths?

More broadly speaking, the DTU calculations, so successful in the hadron sector, should now be extended to all hadrons, and compared with experiment, probably leading to a quantitative phenomenological understanding of strong interactions.

In this whole work we have never dealt with electromagnetic and weak interactions; as it stands, our approach deals only with strong interactions. And yet it is in the area of electromagnetic and weak interactions that the conventional constituent quark (parton) approach has met with some of its more impressive successes. Therefore, until the ordered S-matrix approach has been extended to these interactions, and the partonlike features such as scaling understood on that basis, the idea of quarks as constituent particles will probably be hard to dispel completely. To achieve this thus appears as one of the major tasks ahead.

Maybe the most challenging and important problem may turn out to be the development of an appropriate conceptual and mathematical framework in which to formulate the theory of ordered processes. We wish to remind the reader that we have chosen the language of S-matrix theory for this purpose simply because it is the best one available at present; but we should be aware that the conceptual apparatus of quantum mechanics, and of S-matrix theory in particular, is set up to describe statistical correlations between experimental observations, and is hence formulated in terms of the observable parameters momentum and helicity. In particular, it already presupposes various aspects of physical reality, such as a $3 + 1$-dimensional space-time with Minkowski metric, the existence of particles, etc., thus precluding any deeper understanding of these features. On the other hand, ordered processes may operate at another level that is "prior" to space-time. Accordingly, we suspect that in its final form the theory of ordered processes will be formulated in a conceptual and mathematical framework that is yet to be developed, and in which particle order, vertices, color, flavor, mates, etc., would find their natural interpretation. And in this framework questions like why space is three-dimensional (this might well be related to the threeness of color and the resulting fundamental role of the three-vertex), the origin of space-time, and other basic questions might be susceptible to inquiry.

We find this promise of a radical conceptual revolution as exciting as the prospect of a phenomenologically successful theory of strong interactions. In the following postscript we engage in some rather general speculation on the direction that this development could take.

PHILOSOPHICAL POSTSCRIPT

Classical physics is closely associated with the mechanistic philosophy of nature according to which the objective reality of the universe is such that it can be described as a space-time continuum containing material objects analyzable into indivisible, elementary constituent particles, and electromagnetic fields, both of which evolve in time according to deterministic differential equations of motion; all other quantities and qualities can in principle be reduced to these basic concepts.

This view of the world resulted from claiming absolute and unlimited validity for a set of concepts and relationships that has only approximate validity in a limited domain. And although this kind of sweeping generalization could not conceivably be considered scientific, it nevertheless became the generally accepted "scientific world-view," *the* scientific paradigm, governing not only physics but all of natural science and increasingly even social sciences and humanities, limiting the possible forms of models and theories scientists were willing to consider, the kinds of connections and relationships they were prepared to look for, and even the kinds of phenomena they considered possible or "legitimate."

With the advent of quantum theory the limitations of classical physics became apparent, and the rug was completely swept from under this philosophy of nature. Not only did determinism have to be discarded but it was recognized that *any* model of reality based on the traditional notion of localized objects existing in space-time necessarily contradicted the experimental predictions of quantum theory (Stapp, 1971a; Bell, 1978; Clauser and Shimony, 1978); that space and time could not be considered as objective realities, but rather as categories of our experience [much like color, which already classical physics had reduced from the objective property the naive view holds it to be to something "in the eye of the beholder" (Stapp, 1971b)]; that particles did not have the attributes of material objects, but rather of relationships (Stapp, 1971a); that the idea of "composite particles" versus "elementary particles" (e.g., that a hydrogen atom "consisted of" a proton and an electron) is a nonrelativistic approximation that breaks down at the level of strong interactions (Blancenbecler et al., 1960). Thus all the elements of the traditional world-view have been discredited as fundamental notions, even if they retain their usefulness in the domain of everyday experience from which they are drawn.

Nevertheless, 50 years after the discovery of quantum theory, the view of the world and of man based on classical physics still dominates the minds of most scientists, including physicists; e.g., particles may be imagined as essentially pointlike objects, but with an intrinsic uncertainty in position and momentum due to the impossibility of measuring both quantities simultaneously; or they may be imagined as waves collapsing with each subsequent measurement.

The main reason for this persistence of classical intuition is presumably the fact that quantum physics, unlike classical physics, does not admit a realistic interpretation (Stapp, 1971b, 1970): it cannot be interpreted as describing an objective reality rooted in space-time, and indeed does not refer to any reality at all; all attempts at realistic interpretations of quantum theory have ended in intractable paradoxes. Instead, quantum theory has to be interpreted pragmatically[2] (Copenhagen interpretation): it provides statistical correlations between sets of observations (preparation and measurement) (Stapp, 1971a). Since quantum theory does not give us a new picture of reality to hold on to, it is understandable that the old, fundamentally incorrect but practically often useful pictures have survived so long.

An attempt to develop a new model of reality compatible with relativistic quantum theory was undertaken by H. P. Stapp (1975), based on a philosophical framework constructed by A. N. Whitehead (1979). In this model, physical reality is conceptualized as a sequence of events that can be localized (assigned a location in a four-dimensional mathematical space). Quantum mechanics (*S*-matrix theory) represents the "pragmatic limit" of this model, i.e., its statistical predictions for a scattering experiment.

We suspect that it is in this context that particle order must be seen: the process graphs symbolize patterns of causal connectedness between events.

[2] For an introduction to pragmatism see James (1970).

Thus the picture of the material universe suggested here is one of a web of relationships rather than one of a collection of material objects. Interestingly enough, this view bears a strong similarity to various Eastern philosophies (Taoism, Buddhism and others) (Capra, 1975), just as the materialist philosophy of classical physics bears a strong resemblance to the atomistic philosophy of Democritus.

We believe that science, with physics as usual in the vanguard, is about to undergo a major paradigm shift in the sense outlined above, and that particle order could prove an ingredient in bringing it about and shedding new light on the nature of physical reality.

ACKNOWLEDGMENTS

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Numerous conversations with Phil Lucht, Yoav Eylon, and Fritjof Capra also played a useful role in this development.

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[1] H. P. Stapp has arrived at these same graphs using a different approach (Stapp, 1977a, b).

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